Cosmology "Summary"

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1 Fundamental Questions

Cosmology is the study of large and small structures of the Universe.
Cosmology is an unique science:

- There is only one (visible) Universe to observe
- For the first time in human history, we are able to address the great questions of Cosmology
- On cosmological scales, the finite velocity of light becomes a critical factor. Thus, it also looks back in time, to the earliest moments, making it the ultimate archaeological science.
- We read of the sound spectrum of the Universe.

1.1 Age of Precision Cosmology

Since less than 20 years ago we live in the **Age of Precision Cosmology**, we know a lot of factors of the Universe very well:

- The Universe was formed in the Hot Big Bang, this was $T_0 = 13.798 \pm 0.037$ gigayears ago.
- Space has been expanding ever since, with a rate of $H_0 = 67.74 \pm 0.46$ km/s/Mpc.
  - This expansion has been accelerating since 6.7 ± 0.4 gigayears ago.
- The Universe has and average energy density of $\Omega_0 = 0.862 \times 10^{-29}$ g/cm^3.
- The average temperature of the Universe is 2.72548 ± 0.00057 K.
  - The temperature/density fluctuations over the Universe are of a magnitude $\Delta T/T < 10^{-5}$.
- The radius of the observable Universe is $d_H \sim 41$ giga lightyears. Within this horizon there are:
  - approximately 100×10^9 galaxies,
  - approximately 200×10^{18} stars.
- For every atom in the Universe there are 1.9×10^9 photons (n_γ/n_b ratio).
- Space is almost completely flat: $\Omega_k = 0.000 \pm 0.005$.
- The cosmic composition of the Universe is as follows:
  - $\sim 4.9\%$ Baryonic Matter (protons/neutrons/more particles),
  - $\sim 26.8\%$ Dark Matter,
  - $\sim 63.8\%$ Dark Energy.

Radius of the observable Universe. How can the radius of the observable Universe be $d_H \sim 41$ giga lightyears? You'd expect it to be 13.798 ± 0.037 giga lightyears. Well, light propagates with respect to space, but the expansion of the Universe is nothing but the stretching of space, so the light moves through space that is being stretched (think of a rubber band being stretched).

1.2 What Questions Does Cosmology Try to Answer?

1.2.1 Does the Universe have an origin? If so, how old is it?

In an infinitely large, old and unchanging Universe each line of sight would hit a star, then the night sky would be as bright as surface of star. This is not the case, thus the Universe has a finite age; 13.798 ± 0.037 gigayears. This is known as **Olbers' paradox**.

1.2.2 What is the fate of the Universe?

1.2.3 What are the components of the Universe ?

How does each influence the evolution of the Universe ?

How is each influenced by the evolution of the Universe ?

1.2.4 How "big" is the Universe? (finite, infinite?)

The radius of the observable Universe is $d_H \sim 41$ giga lightyears. This is the Horizon of the Universe:

$$R_{\text{Hor}} = a(t) \int_0^t \frac{c\,dt'}{a(t')} = 41.1 \text{ giga lightyears}$$

The horizon of the Universe is distance that light travelled since the Big Bang. $a(t)$ is the accelration factor of the expansion.

1.2.5 What is the role of humans in the cosmos?

Humans are definitely not the centre of the Universe as some religions might make it out to be, but we're not insignificant. It is incredible that the laws of Physics, Chemistry and Biology just happen to create live, create us humans and that we're able to understand and decipher these laws of nature.

1.2.6 Unanswerable Questions (At This Moment)

Is our Universe unique, or are there many other Universes (multiverse)?

This gets closer to the realm of metaphysics, these other Universes would never be visible to us.

What made the Universe originate?

We know how, but why? this we don't know because we can't see that, we can't say anything with any certainty of what happened before Planck time.

Why are the physical laws as they are ?
Do they need to be?

We are able to find out how thing happen and describe this with physical laws, but we don't know why these laws are what they are. **How many dimensions does the Universe have?**
More than 1timelike + 3 spacelike?

String theories predict the possibility of 11 dimensions.

Are our brains sufficiently equipped to understand and answer the ultimate questions? For example, a cat can be very smart for a cat, but it will never be able to understand quantum physics, because their brain just isn't capable of it.

2 Gravity

The weakest fundamental force is gravity, however, its range is infinite and not shielded. It is cumulative as all mass adds, while for example electromagnetic charges can be + or -, cancelling each others effect. Because of this it dominates the Universe.

2.1 Four Fundamental Forces of Nature

Four types of interaction field may be distinguished in nature:

- Strong Nuclear Force
  - Responsible for holding particles together inside the nucleus.
- Electromagnetic Force
  - Responsible for electric and magnetic interactions, and determines structure of atoms and molecules.
- Weak Force
  - Responsible for (beta) radioactivity.
- Gravity
  - Responsible for the attraction between masses.
  - By far the weakest force of nature.

	Relative strength	Range
Strong field	$\sim 10^{38}$	Short, $\sim$1 fm= 10^{-15} m
Electromagnetic field	$\sim 10^{36}$	Long, $\propto \frac{1}{r^2}$
Weak field	$\sim 10^{25}$	Short, $\sim 10^{-2}$ fm= 10^{-17} m
Gravitational field	1	Long, $\propto \frac{1}{r^2}$

The particles associated with the interaction fields are bosons.

Force	Carried by	Acts on
Strong field	Gluon	Quarks and Gluons
Electromagnetic field	Photon	Quarks, Charged leptons, W^+, W^-
Weak field	W^+, W^-, Z^0	Quarks and Leptons
Gravitational field	Graviton (not observed yet)	All

2.2 Newton's Static Universe

2.2.1 Newton's Laws of Motion

Newton's 1st Law. zero force - body keeps constant velocity

$$\mathbf{F} = 0 \Rightarrow \mathbf{v} = \text{constant}$$

Newton's 2nd Law. force = acceleration x mass = change of velocity x mass

$$\mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt}$$

Newton's 3rd Law. action = reaction

$$\mathbf{F}_a = -\mathbf{F}_b$$

2.2.2 Newton's Gravity

In the early 1600s, Isaac Newton developed his law of gravity, showing that motion in the heavens obeyed the same laws as motion on Earth.

$$\mathbf{F}_g = -G\frac{mM}{r^2}\mathbf{e}_r$$

Some observations started to hinder this formula, it was not possible to explain the motion of the planet Mercury.
Thus, Newton ran into trouble when he tried to apply his theory of gravity to the entire universe. His law predicted that all the matter in the universe should eventually clump into one big ball. Newton knew this was not the case, and assumed that the universe had to be static:

'the Creator placed the stars such that they were "at immense distances from one another."'

2.2.3 Implications of Newton's Universe

- Absolute and uniform time
- Space & time are independent of matter and also independent of each other
- Dynamics:
  - action at distance (there is some mysterious force that affects everything)
  - instantaneous (this force acts instantaneously at any distance)
- Universe edgeless, centerless & infinite
- Cosmological Principle: *'Universe looks the same at every place in space, every moment in time'*
- Absolute, static & infinite space

2.3 Einstein's Dynamic & Geometric Universe

- General Relativity is a "metric theory": gravity is a manifestation of the geometry, curvature, of space-time.
- Revolutionized our thinking about the nature of space & time:
  - no longer Newton's static and rigid background,
  - a dynamic medium, intimately coupled to the universe's content of matter and energy.

2.3.1 Einstein's Field Equations

$$R^{\alpha\beta} - \frac{1}{2}Rg^{\alpha\beta} = -\frac{8\pi G}{c^4}T^{\alpha\beta}$$

The left hand side describes curvature in 16 numbers, α and β are the space time dimensions $(0, 1, 2, 3$ for $t, x, y, z)$. This gives basically 10 equations because of symmetry.
The right hand side: T stands for energy and momentum.

At some limits Einstein's theory and Newton's theory will of course come to the same conclusions.

2.3.2 Implications of Einstein's Universe

- Spacetime is dynamic
- Local curvature & time determined by mass
- Bodies follow shortest path through curved spacetime (geodesics)
- Dynamics:

   - action through curvature space
   - travels with velocity of light

2.4 Newton vs Einstein

In short:

10 field equations	1 field equation
10 potentials	1 potential
nonlinear equations	linear equations
intrinsically geometric	absolute space and time
can cope with infinite space	requires finite space
all energies gravitate	gravitation mass-density only
pressure gravitates	gravitation mass-density only
Cosmological Constant feasible	repulsive action gravity impossible
Hyperbolic propagation	instantaneous propagation
Singularities spacetime	Singularities space
Horizons & Black Holes	No Horizons & Black Holes
Gravitational Waves	No Gravitational Waves

2.5 Relativity: Space & Time

2.5.1 Fundamental Relativity Tenets.

- All **Laws of Nature** are equivalent in **reference frames** in uniform relative motion
- The (Vacuum) **speed of light** is $c = 3 \times 10^8$ m/s in all such frames

2.5.2 Relativistic Spacetime.

Having a constant speed of light in all reference systems is only possible if time and space are not absolute, but dependent on the reference system. This leads to:

- Time dilation
- Length contraction
- Relativity of Simultaneity

A Lorentz transformation describes the relation between two reference frames.

Time Dilation and Length Contraction. Imagine you're shining a light into a mirror. The time interval measured in the frame of a train train passenger would be:

$$\Delta t_1 = \frac{\text{round trip distance}}{\text{speed of light}} = \frac{2d}{c}$$

An observer in a reference frame on the train platform would measure a longer time:

$$\Delta t_2 = \frac{2\sqrt{d^2 + (v\Delta t_2/2)^2}}{c}$$
$$\Delta t_2 = \frac{1}{\sqrt{1-(v/c)^2}}\left(\frac{2d}{c}\right) = \gamma \Delta t_p$$

With the Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} > 1$$

At the same time, we notice the effect on the other component of spacetime:

$$L = L_0\sqrt{1-\frac{v^2}{c^2}} = \frac{L_0}{\gamma}$$

Thus an observer will note

- a slowing of clocks (time dilation)
- a shortening of rulers (length contraction)

that are moving with respect to the observer. This effect becomes significant only if the clock or ruler is moving at a substantial fraction of the speed of light.

2.5.3 Minkowski Spacetime.

A point in 4D spacetime has the coordinates:

$$\mathbf{x}^\mu = (ct, x, y, z)$$

thus:

$$\mathbf{x}^0 = ct$$
$$\mathbf{x}^1 = x,\ \mathbf{x}^2 = y,\ \mathbf{x}^3 = z$$

Distances in flat (Minkowski) spacetime are given by:

$$s^2 = c^2\boldsymbol{t}^2 - \boldsymbol{x}^2 - \boldsymbol{y}^2 - \boldsymbol{z}^2$$

Using tensor calculus and the Einstein summation convention this becomes:

$$s^2 = \eta_{\mu\nu}\boldsymbol{x}^\mu\boldsymbol{x}^\nu$$

with $\eta_{\mu\nu}$ the metric tensor for Minkowski space:

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

2.5.4 Lorentz Transformation

The Lorentz transformation between reference frames is give by:

$$x' = \gamma_v\left(x - \frac{v}{c}ct\right)$$
$$y' = y$$
$$z' = z$$
$$ct' = \gamma_v\left(ct - \frac{v}{c}x\right)$$

with

$$\gamma_v = \frac{1}{\sqrt{1-(v/c)^2}} > 1$$

Using tensor calculus and the Einstein summation convention this becomes:

$$\boldsymbol{X}'^\mu = \Lambda^\mu{}_\nu\boldsymbol{x}^\nu$$

with the **Lorentz transform tensor**:

$$\Lambda^{\mu}{}_{\nu} = \begin{pmatrix} \gamma & \beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The interval is invariant under Lorentz transform

$$c^2 d\tau^2 = c^2 dt^2 - \left(dx^2 + dy^2 + dz^2\right)$$

2.5.5 Frames of Reference

Proper Time, the time of the "acting" reference frame, is given by:

$$d\tau = dt \left(1 - \frac{v^2}{c^2}\right)^{1/2}$$

2.5.6 Relativistic Mechanics

There are more important four-vectors, like spacetime. The four-velocity four-vector is given by:

$$U^{\mu} = \frac{dx^{\mu}}{d\tau} = \gamma \begin{pmatrix} c \\ \mathbf{u} \end{pmatrix}$$

and

$$U^{\mu} U_{\mu} = -c^2$$

The four-momentum four-vector is given by:

$$P^{\mu} = m_0 U^{\mu} = \begin{pmatrix} E/c \\ \mathbf{p} \end{pmatrix}$$

and

$$P^{\mu} P_{\mu} = -\frac{E^2}{c^2} + |\mathbf{p}|^2$$

and the energy of a particle at rest of course follows from the four-momentum:

$$E = m_0 c^2$$

2.6 Relativity: Curved Space

2.6.1 Inertial vs Gravitational Mass

A larger mass experiences a stronger gravitational force than a light mass (gravitational mass). But a larger mass is more difficult to get moving than a light mass (inertial mass). As a result, a heavy mass falls equally fast as the light mass, thus:

Gravitational Mass = Inertial Mass

2.6.2 The Equivalence Principle

There is no experiment that can distinguish between uniform acceleration and a uniform gravitational field. From this principle it follows that free-falling bodies follow straight worldlines (geodesics) in curved spacetime.

2.6.3 Curvature and Metric

Distances in curved spacetime:

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$$

with $g_{\alpha\beta}$ the metric tensor for curved spacetime: metric specifies distance recepy.
Proper time is given by:

$$d\tau = dt\sqrt{1 + \frac{2\phi}{c^2}}$$

From Equivalence Principle, one may derive the equation of motion:

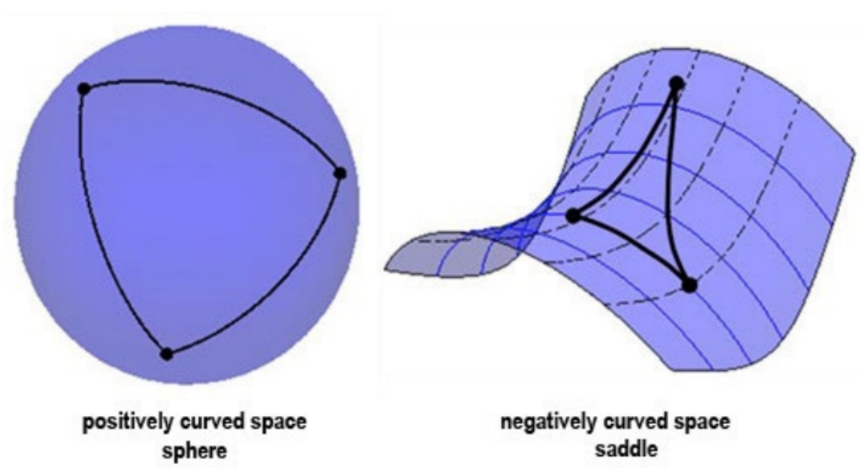
$$\frac{d^2 x^\beta}{d\tau^2} + \Gamma^\beta_{\lambda\nu} \frac{dx^\lambda}{d\tau} \frac{dx^\nu}{d\tau} = 0$$

with **Christoffel symbol**/connection

$$\Gamma^\alpha_{\beta\gamma} = \frac{1}{2} \boldsymbol{g}^{\alpha\nu} \left\{ \frac{\partial \boldsymbol{g}_{\gamma\nu}}{\partial \boldsymbol{x}^\beta} + \frac{\partial \boldsymbol{g}_{\beta\nu}}{\partial \boldsymbol{x}^\gamma} - \frac{\partial \boldsymbol{g}_{\gamma\beta}}{\partial \boldsymbol{x}^\nu} \right\}$$

This is actually exactly the same equation as that for shortest paths in general curved spaces (in 4D spacetime), the GEODESIC equation.

Curved Space. Space can be positively and negatively curved:



Triangle angles $>$ 180 degrees
Circle circumference $< 2\pi r$

Triangle angles $<$ 180 degrees
Circle circumference $> 2\pi r$

2.7 Einstein Field Equation

2.7.1 Gravity & Curved Spacetime

Fundamental tenet of General Relativity:

Gravity is the effect of curved spacetime!

E.g.: relation between metric component g_{00} and gravitational potential

$$g_{00} = \frac{2\phi}{c^2}$$

2.7.2 Source of Gravity: Energy-Momentum Tensor

Energy and **momentum** are intimately linked physical quantities: both components of the **energy-momentum** four-vector:

$$P^{\mu} = m_0 U^{\mu} = m_0 \gamma \begin{pmatrix} c \\ \mathbf{u} \end{pmatrix} \approx m_0 \begin{pmatrix} c \\ \mathbf{u} \end{pmatrix}$$

Relativity is concerned with formulating physical laws and relations in a **coordinate-free form**. This is called formulation in **covariant form**: looking for tensor equations that are valid in any reference frame.

Now we have the energy momentum-tensor (specifying the energy & momentum content of the Universe):

$$T_{\mu v} = \left(\rho + \frac{p}{c^2}\right) U^{\mu} U^{v} - p g^{\mu v}$$

Its covariant form, spatial derivative, is zero:

$$T_{\mu\nu;\nu} = 0$$

In a restframe this tensor is:

$$T_{\mu\nu} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

Notice the presence of the pressure term:

- pressure is the flux of momentum
- In relativity momentum is coupled to energy in energy-momentum four-vector

In cosmology this becomes of KEY significance: source term of gravity: Energy & Momentum, hence Energy & Pressure appear in dynamics.

$$\nabla^2 \phi = 4\pi G \left(\rho + \frac{3p}{c^2}\right)$$

2.7.3 Einstein Field Equation

The Einstein tensor describes the curvature of spacetime:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

($R_{\mu\nu}$ is the Ricci tensor, the curvature of space and the metric $g_{\mu\nu}$ is a measure of the distance.)
The Einstein tensor's covariant form, spatial derivative, is also zero:

$$G_{\mu\nu;\nu} = T_{\mu\nu;\nu} = 0$$

Then these two tensors need to be proportional to each other (because of tensor calculus):

$$G_{\mu\nu} \propto T_{\mu\nu}$$

This then gives:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

The metric tensor ALSO has a covariant derivative that is zero, this gives the freedom to add a multiple of metric tensor to Einstein tensor:

$$G_{\mu v} + \Lambda g_{\mu v} = -\frac{8\pi G}{c^4} T_{\mu v}$$

where Λ is the Cosmological Constant (dominant term). This is the **Einstein Field Equation**.
This equation describes on the curvature of spacetime on the lhs says that it is curved because of the energy-momentum described on the rhs.

Spacetime reacts to the content of the Universe $\Longleftrightarrow$ The content of the Universe reacts to the curvature

Rewriting this gives:

$$G^{\mu v} = -\frac{8\pi G}{c^4}\left(T^{\mu v} + T^{\mu v}_{vac}\right)$$
$$T^{\mu v}_{vac} \equiv \frac{\Lambda c^4}{8\pi G} g^{\mu v}$$

Where we can call $T^{\mu v}_{vac}$ Dark Energy, here we thus make the cosmological constant part of the energy side of the equation, in the other case it is part of the curvature side.

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4}T_{\mu\nu}$$
$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -\frac{8\pi G}{c^4}T_{\mu\nu} - \Lambda g_{\mu\nu}$$

The left hand side describes curvature, the right hand side describes energy and momentum.

3 The Cosmological Principle and Friedman-Lemaitre Universe

The Einstein Field Equations are notoriously complex, essentially 10 equations. Solving them for general situations is almost impossible. However, there are some special circumstances that do allow a full solution. The simplest one is also the one that describes our Universe.

3.1 The Cosmological Principle

The **cosmological principle** is usually stated formally as *"Viewed on a sufficiently large scale, the properties of the universe are the same for all observers"*.

It describes the symmetries in global appearance of the Universe:

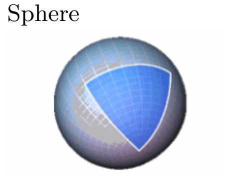
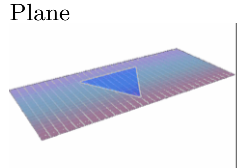
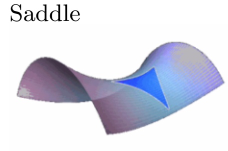
- Homogeneous, the Universe is the same everywhere
- Isotropic, the Universe looks the same in every direction
- Universality, physical laws are the same everywhere
- Uniformly Expanding, the Universe "grows" with same rate in every direction and at every location

3.1.1 Geometry of the Universe

There exist no more than THREE *uniform* spaces:

1) Euclidian (flat) Geometry	Euclides
2) Hyperbolic Geometry	Gauß, Lobachevski, Bolyai
3) Spherical Geometry	Riemann

(Others are *not* uniform)

Property	Closed (Spherical)	Euclidean	Open (Hyperbolic)
Spatial Curvature	Positive	Zero	Negative
Circle Circumference	$< 2\pi R$	$2\pi R$	$> 2\pi R$
Sphere Area	$< 4\pi R^2$	$4\pi R^2$	$> 4\pi R^2$
Sphere Volume	$< \frac{4}{3}\pi R^3$	$\frac{4}{3}\pi R^3$	$> \frac{4}{3}\pi R^3$
Triangle Angle Sum	$> 180^o$	180^o	$< 180^o$
Total Volume	Finite ($2\pi^2 R^3$)	Infinite	Infinite
Surface Analog	Sphere	Plane	Saddle
			

Lecture 3a

3.2 The Cosmological Principle

Before embarking upon an assessment of the reality of the **homogeneity** and **isotropy** of our Universe, we first should pay attention to the meaning of these two concepts:

- **Homogeneity**, the same physical circumstances prevail at any location in the Universe. In its broadest context this means:
  - physical quantities like density, temperature, pressure, ...
  - physical laws and relations
- **Isotropy**, the Universe looks the same in whatever direction you look.

It is important to note that:

- **Homogeneity** does not imply **Isotropy**
  In principle it is perfectly possible to have an anisotropic & homogeneous medium:
  - eg. more stretched in one direction than in the other, nonetheless exactly the same at every point in space:
  - the local anisotropy may be the same everywhere
- **Isotropy** does not imply **Homogeneity**
  Given we know the Universe only from one vintage point, our cosmic location, we cannot be sure that our conclusion of isotropy is universilly valid:
  - however, it violates our Copernican feelings
  - however, if you would know isotropy to hold for 2 locations, homogeneity and isotropy holds throughout the Universe

While the "**homogeneity**" and "**isotropy**" of the Universe at first sight might occur like rather crude approximations of reality, there is ample evidence that on scales exceeding a few hundred Mpc it is a reasonably accurate description of reality.

3.3 Prime Evidence for an Isotropic Universe:

3.3.1 Cosmic Microwave Background

Cosmic Microwave Background is Thermal radiation pervading throughout the whole Universe. It has a temperature of $T_\gamma = 2.725$ K. By far CMB photons represent the most abundant species in the Universe.
CMB is isotropic to almost absurdly accurate levels, it is all around the same temperature, thus this is the primary evidence for an isotropic Universe.

$$\frac{\Delta T}{T} < 10^{-5}$$

It is the most accurately known Black Body radiation field in nature.

3.3.2 X-ray Background

The sky glows in X-rays in every direction, the X-ray background, coming from all the galaxies in the Universe. If you were to see more galaxies in one direction than another direction, the X-ray background wouldn't be isotropic, but the X-ray background is highly isotropic. At least 80 percent of diffuse hard X-ray background resolved into very many very faint (and distant) sources.

3.3.3 Gamma Ray Bursts (GRBs)

Gamma Ray Bursts are brilliant flashes of γ-ray emission, from around 1 msec to several 10s of seconds followed by afterglows of X-ray, optical & radio emission. Originally it was thought the came from the solar system of from galaxies, but this wasn't the case.
Most energetic events since Big Bang, 10^{54} ergs at peak emission it is in the order of the energy of all galaxies in visible Universe.
Two classes GRBs:

- long duration, more than 2 sec., caused by collapsar/hypernovae
- short duration, between msec and 2 sec., caused by neutron star – neutron star mergers

(short duration GRBs 10 times less bright than long duration ones)

3.3.4 Galaxy Sky Distribution (on cosmologically relevant scales)

Although we know that the local Universe is far from isotropic, or homogeneous, when assessing the galaxy distribution to high depths, there is a high level of isotropy.

Example of a "nearby" galaxy sample:
clear demonstration increasing isotropy for higher/fainter levels

Example of "extended nearby" galaxy sample:

- counting 2 million galaxies up to m=20.5
- local "Cosmic Web" is visible as tenuous filamentary traces and compact dense cluster nodes
- most of the sky marked by a uniform distribution of background galaxies
- on scales $>$ 100 Mpc hardly any noticeable structure

Even more compelling is the evidence from deep "radio galaxy" samples:
almost perfectly isotropic

3.3.5 Hubble Expansion

Global Hubble Expansion Observations over large regions of the sky, out to large cosmic depth:

- the Hubble expansion offers a very good description of the actual Universe
- the Hubble expansion is the same in whatever direction you look: isotropic

3.4 Prime Evidence for an Homogeneous Universe:

3.4.1 The spatial galaxy distribution in galaxy redshift surveys

A clear obvious test for homogeneity of the Universe is to make a map of the matter distribution in the Universe and identify:

Whether there is structure:
- yes, obvious: stars, galaxies, clusters, superclusters

What the size is of the largest structures in the Universe, ie.
- is there a scale beyond which the Universe tends to converge to uniformity
- or, do you see ever larger structures as you probe deeper into the Universe

The best probe for studying the Universe is (still) the distribution of galaxies

Cautious Conclusion:
While galaxy redshift surveys mapped ever larger regions of the nearby Cosmos and revealed outstanding and intriguing structures got revealed, there is no indication for large inhomogeneities on scales bigger than

150-200 h^{-1} Mpc. Thus the assumption of global cosmological homogeneity appears to be corroborated by our "cosmographic" maps.

3.4.2 Galaxy number counts

Number counts of galaxies, ie. the number of galaxies with apparent magntiude m or higher in a certain region of the sky, contain potentially a large of amount of information on the structure of the Universe. Hubble used counts of galaxies to demonstrate that the distribution of galaxies is homogeneous on the largest cosmological scales.

Using the assumption of homogeneity, the number of sources brighter than flux S would be:

$$N(S) \approx S^{-3/2}$$

Which in terms of magnitude m, translates into

$$N(m) \approx 10^{0.6m}$$

Number counts in many galaxy surveys indeed reveal the expected scaling at (relatively) bright magnitudes.

3.4.3 Scaling of the Projected galaxy distribution

Galaxy sky distribution:
Galaxies clustered, a projected expression of the true 3-D clustering

Probability to find a galaxy near another galaxy higher than average (Poisson) probability

Quantitatively expressed by 2-pt correlation function $\omega(\theta)$:

$$dP(\theta) = \overline{n}^2(1 + \omega(\theta))d\Omega_1 d\Omega_2$$

Excess probability of finding 2 galaxies at angular distance θ. It gives the strength of the "clumpyness" as a function of angular distance. Clumpyness gets less at larger distances.

This is clear evidence that there are no significant large structures on scales larger then 100-200 Mpc.

3.4.4 Cosmic Dipole

The sky map of the CMB has a clear dipole anisotropy. It has an amplitude of 3.4 mK. It's a manifestation of Doppler shift CMB radiation: result our motion with respect to the Universe.
It is the result of the gravitational attraction by surrounding matter concentrations. The dipole motion is embedded within a coherent shear flow, largely in the direction of the so called "Great Attractor".

Lecture 3, Continued

3.5 Robertson-Walker Metric:

Distances in a uniformly curved spacetime is specified in terms of the Robertson-Walker metric. The space-time distance of a point at coordinate (r, θ, ϕ) is:

$$ds^2 = c^2 dt^2 - a(t)^2 \left\{ dr^2 + R_c^2 S_k^2 \left(\frac{r}{R_c} \right) \left[d\theta^2 + \sin^2 \theta d\phi^2 \right] \right\}$$

Where the function $S_k(r/R_c)$ specifies the effect of curvature on the distances between points in spacetime:

$$S_k \left(\frac{r}{R_c} \right) = \begin{cases} \sin \left(\frac{r}{R_c} \right) & k = +1 \\ \frac{r}{R_c} & k = 0 \\ \sinh \left(\frac{r}{R_c} \right) & k = -1 \end{cases}$$

With $k = +1$ being an universe with a spherical geometry (positive curvature), $k = 0$ being an universe with Euclidian (flat) geometry (zero curvature) and $k = -1$ being an universe with an hyperbolic geometry (negative curvature).

For a flat Universe:

$$\text{(Euclidean): } dx^2 = dr^2 + r^2\left[d\theta^2 + \sin^2\theta\, d\phi^2\right]$$
$$\text{(Minkowski space with expansion): } ds^2 = c^2dt^2 - a(t)^2\left\{dr^2 + r^2\left[d\theta^2 + \sin^2\theta d\phi^2\right]\right\}$$

3.6 Friedmann-Robertson-Walker-Lemaître (FRLW)

The Einstein Field Equation is given by:

$$G_{\mu\nu} = -\frac{8\pi G}{c^4}T_{\mu\nu}$$

We know that the Einstein Field Equation contains 16 equations, but due to symmetry this leads to 10 equations. However, 8 of the equations depend on the 2 other (independent!) equations. These two are surprisingly simple and are given here:

$$G^0_0 \quad \rightarrow \quad G^0_0 = 3\left(\dot{R}^2 + kc^2\right)/R^2 = \frac{8\pi G}{c^2}\rho c^2$$
$$G^1_1 \quad \rightarrow \quad G^1_1 = \left(2R\ddot{R} + \dot{R}^2 + kc^2\right)/R^2 = -\frac{8\pi G}{c^2}p$$

Combining these two equations gives:

$$\ddot{R} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)R + \frac{\Lambda}{3}R \text{ (acceleration of the radius of curvature)}$$
$$\dot{R}^2 = \frac{8\pi G}{3}\rho R^2 - kc^2 + \frac{\Lambda}{3}R^2 \text{ (something to do with energy)}$$

3.6.1 Cosmic Expansion Factor

The cosmic expansion factor is given by:

$$a(t) = \frac{R(t)}{R_0}$$

The current expansion factor is:

$$a(t) = \frac{R_0}{R_0} = 1$$

The Cosmic Expansion is a uniform expansion of space:

$$\mathbf{r}(t) = a(t)\mathbf{x}$$

3.6.2 Friedmann-Robertson-Walker-Lemaître Universe

Together this gives the **Friedmann-Robertson-Walker-Lemaître equations**:

$$\ddot{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)a + \frac{\Lambda}{3}a$$
$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - \frac{kc^2}{R_0^2} + \frac{\Lambda}{3}a^2$$

These equations have four important factors, that apparently fully determine the evolution of the Universe:

Density:	$\rho(t)$
Pressure:	$p(t)$
Curvature:	kc^2/R_0^2 – $k = -1, 0, +1$ – R_0 is the present curvature radius
Cosmological constant:	Λ

- Density & Pressure:
  - In relativity, energy & momentum need to be seen as one physical quantity (four-vector)
  - Pressure is the momentum flux
- Curvature:
  - Gravity is a manifestation of geometry spacetime
  - Doesn't exist in Newton's gravity
- Cosmological Constant:
  - Free parameter in General Relativity
  - Einstein's "biggest blunder"
  - Mysteriously, since 1998 we know it dominates the Universe
  - Does definitely not exist in Newton's gravity

Pressure, **curvature** and the **cosmological constant**, which are found in Relativistic Cosmology are *never* found in Newtonian Cosmology, this mainly deals with **energy**.

3.7 Hubble(-Lemaître) Parameter

Cosmic Expansion is a uniform expansion of space. Objects do not move themselves, they are like beacons tied to a uniformly expanding sheet.

$$\left.\begin{array}{l}\mathbf{r}(t) = a(t)\mathbf{x} \\ \dot{\mathbf{r}}(t) = \dot{a}(t)\mathbf{x} = \frac{\dot{a}}{a}a\mathbf{x} = H(t)\mathbf{r}\end{array}\right\} H(t) = \frac{\dot{a}}{a}$$

Here $H(t)$ is the Hubble parameter, the Hubble "constant" is given by $H_0 = H(t = t_0)$.
For a long time, the correct value of the Hubble constant H_0 was a major unsettled issue, distances and timescales in the Universe had to deal with uncertainties of a factor 2. Now the Hubble constant is measured to be $H_0 = 71.9$ km/s/Mpc.

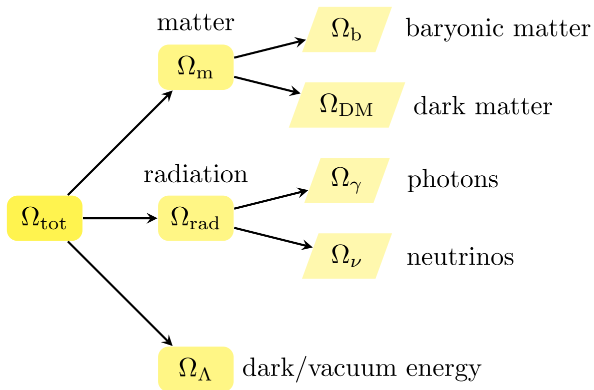
However, "late universe" measurements using calibrated distance ladder techniques have converged on a value of approximately 73 km/s/Mpc. "Early universe" techniques based on measurements of the cosmic microwave background have become available, and these agree on a value near 67.7 km/s/Mpc. As techniques have improved, the estimated measurement uncertainties have shrunk, but the range of measured values has not, to the point that the disagreement is now highly statistically significant. This discrepancy is called the **Hubble tension**.

The Hubble time is given by:

$$t_H = \frac{1}{H_0}$$

4 Cosmic Constituents

The total energy content of Universe made up by various constituents, the principal ones being:

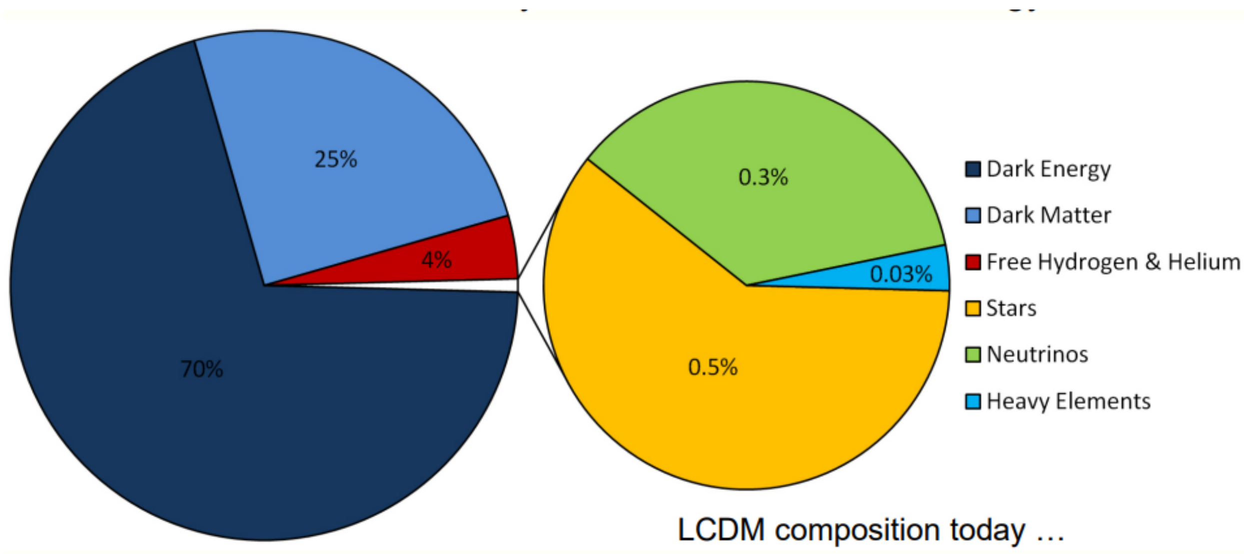


In addition, there are contributions by

- gravitational waves,
- magnetic fields,
- cosmic rays, ...

But there are poor constraints on their contribution: henceforth we will not take them into account.

In percentages these constituents are divided as follows:



So far, we've only seen the white sliver of this energy content.

4.1 Critical Density & Cosmological Density

4.1.1 Critical Density

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - \frac{kc^2}{R_0^2}$$

The **Critical Density** is the density at which the curvature of the Universe is zero. Assuming:

- For a Universe with $\Omega = 1$
- Given a particular expansion rate $H(t)$
- Density corresponding to a flat Universe ($k = 0$)

Gives a critical density:

$$\rho_{\text{crit}} = \frac{3H^2}{8\pi G}$$

In a FRW Universe, densities are in the order of the critical density:

$$\rho_{\text{crit}} = \frac{3H^2}{8\pi G} = 1.8791\, h^2 \times 10^{-29} \text{ g cm}^{-3}$$

4.1.2 Cosmological Density

In a matter-dominated Universe, the evolution and fate of the Universe entirely determined by the (energy) density in units of critical density:

$$\Omega \equiv \frac{\rho}{\rho_{\text{crit}}} = \frac{8\pi G\rho}{3H^2}$$

The individual contributions to the energy density of the Universe can be figured into the Ω parameter:

$$\Omega = \Omega_{\text{rad}} + \Omega_{\text{m}} + \Omega_{\Lambda}$$

With

Radiation: $$\Omega_{rad} = \frac{\rho_{rad}}{\rho_{\text{crit}}} = \frac{\sigma T^4/c^2}{\rho_{\text{crit}}} = \frac{8\pi G\sigma T^4}{3H^2c^2}$$

Matter: $$\Omega_{\text{m}} = \Omega_{\text{Dark Matter}} + \Omega_{\text{Baryonic Matter}}$$

Dark energy/ cosmological constant: $$\Omega_{\Lambda} = \frac{\Lambda}{3H^2}$$

4.1.3 Omega and Curvature

There is a 1-1 relation between the total energy content of the Universe and its curvature. From FRW equations:

$$k = \frac{H^2R^2}{c^2}(\Omega - 1)$$

$\Omega < 1$	$k = -1$	Hyperbolic	Open Universe
$\Omega = 1$	$k = 0$	Flat	Critical Universe
$\Omega > 1$	$k = +1$	Spherical	Close Universe

4.2 Evolving Energy Density

To infer the evolving energy density $\rho(t)$ of each cosmic component, we refer to the **cosmic energy equation**. This equation can be directly inferred from the FRW equations

$$\dot{\rho} + 3\left(\rho + \frac{p}{c^2}\right)\frac{\dot{a}}{a} = 0$$

(Derivation in HW 2, Problem 3(a))

The equation forms a direct expression of the adiabatic expansion of the Universe, ie.

$$\left.\begin{array}{ll} U = \rho c^2 V & \text{Internal energy} \\ V \propto a^3 & \text{Expanding volume} \end{array}\right\} dU = -pdV$$

Thus all the energy needed for the expansion of the Universe, thus the Universe is cooling down as a result of the expansion. From this you can also reason back and conclude that in the past, when the Universe was smaller, it also was hotter and that there was a Hot Big Bang.

4.2.1 Energy Density Equation of State

To infer $\rho(t)$ from the energy equation, we need to know the pressure $p(t)$ for that particular medium/ingredient of the Universe.
To infer $p(t)$, we need to know the nature of the medium, which provides us with the equation of state,

$$p = p(\rho, S)$$

For each of the components of the Universe we get:

4.2.2 Matter

Equation of State. The equation of state for matter is given by:

$$p_{\mathrm{m}}(t) = 0$$

The pressure of all matter is negligible, thus is 0.

Evolution of Energy Density. The relation between ρ and a for matter comes from the following differential equation (derived from the cosmic energy equation):

$$\begin{aligned}
\frac{\dot{\rho}}{\rho} &= -3 \cdot \frac{\dot{a}}{a} \\
\frac{\frac{d\rho}{da}\frac{da}{dt}}{\rho} &= -3 \cdot \frac{\dot{a}}{a} \\
\frac{1}{\rho}\frac{d\rho}{da} &= -3 \cdot \frac{1}{a} \\
&\Rightarrow \ln(\rho) = -3\ln(a) \\
&\Rightarrow \rho \propto a^{-3}
\end{aligned}$$

4.2.3 Radiation

Equation of State. The equation of state for radiation is given by:

$$p_{\mathrm{rad}}(t) = \frac{1}{3}\rho_{\mathrm{rad}}c^2$$

Evolution of Energy Density. The relation between ρ and a for radiation comes from the following differential equation (derived from the cosmic energy equation):

$$\begin{aligned}
\frac{\dot{\rho}}{\rho} &= -4 \cdot \frac{\dot{a}}{a} \\
\frac{\frac{d\rho}{da}\frac{da}{dt}}{\rho} &= -4 \cdot \frac{\dot{a}}{a} \\
\frac{1}{\rho}\frac{d\rho}{da} &= -4 \cdot \frac{1}{a} \\
&\Rightarrow \ln(\rho) = -4\ln(a) \\
&\Rightarrow \rho \propto a^{-4}
\end{aligned}$$

Now, why do we have a power of -4 here? We're looking at quantum particles, photons, their wavelength gets stretched due to the Universe stretching, but their frequency is given by $\frac{c}{\lambda}$, and the energy is given by $h\nu$. Thus energy goes down with $\frac{1}{a}$, and due to volume there is a factor $\frac{1}{a^3}$, thus $\frac{1}{a^4}$ in total.

4.2.4 Dark Energy

Equation of State. The equation of state for Dark Energy is given by:

$$p_{\mathrm{DE}} = w\rho_{\mathrm{DE}}c^2$$
$$\Downarrow \; w = -1$$
$$p_\Lambda(t) = -\rho_\Lambda c^2$$

Now how do we get this? Remember that Λ can be either on the curvature side or the energy-momentum side of the Einstein Field Equation. One can infer the energy-momentum tensor for dark energy.

$$T^{\mu v}_{vac} \equiv \frac{\Lambda c^4}{8\pi G} g^{\mu v} \Rightarrow T^{\mu v}_{vac} \equiv \frac{\Lambda c^4}{8\pi G}\eta^{\mu v}$$

$$\eta^{00} = 1, \quad \eta^{ii} = -1$$

Here that negative pressure already shows up. This gives:

$$\left.\begin{array}{l} T^{00}_{vac} = \rho_{vac}c^2 \\ T^{ii}_{vac} = p \end{array}\right\} \Rightarrow \qquad \begin{array}{l} \rho_{vac}c^2 = \frac{\Lambda c^4}{8\pi G} \\ p = -\frac{\Lambda c^4}{8\pi G} \end{array}$$

Thus the Dark Energy equation of state is given by:

$$p_{\mathrm{vac}} = -\rho_{\mathrm{vac}}c^2$$

The parameterized version is given by:

$$p(\rho) = w\rho c^2$$

w basically is there because DE is poorly understood and it helps keep options open, it is probably very close to -1.

Using this in the second Friedmann equation (cosmic acceleration):

$$\ddot{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)a$$

This gives gravitational repulsion if $w < -\frac{1}{3}$, because then $\ddot{a} > 0$.

Evolution of Energy Density. The relation between ρ and a for Dark Energy is given by:

$$\rho_w(a) = \rho_w(a_0)\, a^{-3(1+w)}$$

For the Cosmological Constant this gives:

$$\Lambda : \quad w = -1 \quad \rho_w = cst.$$

For $-\frac{1}{3} > w > -1$ we get:

$$\rho_w \propto a^{-3(1+w)} \quad 1 + w > 0 \quad \text{decreases with time}$$

And for phantom energy this gives:

$$\rho_w \propto a^{-3(1+w)} \quad 1 + w < 0 \quad \text{increases with time}$$

If this phantom energy increases and increases, it could very well be possible it exceeds the binding energies holding the Universe together dissolving the Universe all together.

Dynamic Dark Energy. There is a possibility that this w is dynamically evolving:

$$w(a) = w_0 + (1-a)w_a \approx w_\phi(a)$$

This then gives:

$$\rho_w(a) = \rho_w(a_0)\exp\left\{-3\int_1^a \frac{1+w_\phi(a')}{a'}da'\right\}$$

4.3 Acceleration Parameter

Cosmic acceleration quantified by means of dimensionless deceleration parameter $q(t)$:

$$q = -\frac{a\ddot{a}}{\dot{a}^2}$$

and

$$\begin{aligned} q &= \frac{\Omega_\mathrm{m}}{2} + \Omega_\mathrm{rad} - \Omega_\Lambda \\ &\approx \frac{\Omega_\mathrm{m}}{2} - \Omega_\Lambda \end{aligned}$$

5 Dynamics of FRWL Universes

We can identify 3 types of Universes:

- Open Universes: the Universe will expand forever ($\Omega_0 < 1$)
- Critical Universes: the density is equal to the critical density and the geometry is flat ($\Omega_0 = 1$)
- Closed Universes: the Universe will stop expanding and re-collapse ($\Omega_0 > 1$)

This all depends on the values of the different omegas.

5.1 Components & Evolution

To see how these different Universes depend on the values of the different omegas, we need to do some derivations:

$$\begin{aligned} \dot{a}^2 &= \frac{8\pi G}{3}\rho a^2 - \frac{kc^2}{R_0^2} + \frac{\Lambda}{3}a^2 \\ \frac{\dot{a}^2}{a^2} &= \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2R_0^2} + \frac{\Lambda}{3} \\ H^2 &= \frac{8\pi G}{3}\left(\rho_{\mathrm{m},0}a^{-3} + \rho_{\mathrm{rad},0}a^{-4} + \rho_{\Lambda,0}\right) - \frac{kc^2}{a^2R_0^2} \end{aligned}$$

Remember that:

$$\rho_{\mathrm{crit},0} = \frac{3{H_0}^2}{8\pi G}, \qquad k = \frac{H^2R^2}{c^2}(\Omega - 1),$$

and,

$$\Omega_{m,0} = \frac{\rho_{m,0}}{\rho_{\mathrm{crit},0}}; \quad \Omega_{\mathrm{rad},0} = \frac{\rho_{\mathrm{rad},0}}{\rho_{\mathrm{crit},0}}; \quad \Omega_{\Lambda,0} = \frac{\rho_{\Lambda,0}}{\rho_{\mathrm{crit},0}}$$

Filling this in gives:

$$\frac{H(t)^2}{H_0^2} = \frac{\Omega_{\mathrm{rad},0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$$

(More detailed derivation in HW 3, Problem 1(a))

We can use $1/\dot{a}$ and write this as:

$$H_0 t = \int_0^a \frac{da}{\sqrt{\frac{\Omega_{\mathrm{rad}\ ,0}}{a^2} + \frac{\Omega_{m,0}}{a} + \Omega_{\Lambda,0} a^2 + (1 - \Omega_0)}}$$

5.2 Specific Solutions

While general solutions to the FRW equations is only possible by numerical integration, analytical solutions may be found for particular classes of cosmologies:

- Single-component Universes:
  - empty Universe
  - flat Universes, with only radiation, matter or dark energy
- Matter-dominated Universes
  - Assume radiation contribution is negligible
  - Zero cosmological constant
  - Matter-dominated, including curvature
- Matter+Dark Energy flat Universe

5.3 Single-component Universes

5.3.1 Einstein-de-Sitter Universe

The Einstein-de Sitter Universe is a model with flat geometry containing matter as the only significant substance.

$$\Omega_{\mathrm{m}} = 1, \qquad \Omega_{\mathrm{rad}} = 0, \qquad \Omega_\Lambda = 0, \qquad k = 0$$

The FRW equations reduce to:

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 = \frac{8\pi G \rho_0}{3}\frac{1}{a}$$

Which gives an expansion factor of:

$$\Rightarrow a(t) = \left(\frac{t}{t_0}\right)^{2/3}$$

The age of the Einstein-de Sitter Universe is:

$$t_0 = \frac{2}{3}\frac{1}{H_0}$$

BE ABLE TO INFER THIS!!

5.3.2 Free Expanding (”Milne”) Universe

The Free Expanding (”Milne”) Universe is an empty Universe with negative curvature (by definition, because $\Omega_{\mathrm{m}} = 0$!).

$$\Omega_{\mathrm{m}} = 0, \qquad \Omega_{\mathrm{rad}} = 0, \qquad \Omega_\Lambda = 0, \qquad k = -1$$

The FRW equations reduce to:

$$\dot{a}^2 = -\frac{kc^2}{R_0^2} = cst$$

Which gives an expansion factor of:

$$\Rightarrow a(t) = \left(\frac{t}{t_0}\right)$$

The age of the Free Expanding Universe is:

$$t_0 = \frac{1}{H_0}$$

(Asymptotic situation)

5.3.3 Expansion Radiation Dominated Universe

The Expansion Radiation Dominated Universe is a model of the very early Universe, where the energy density is completely dominated by radiation. The dynamics of the very early Universe is therefore fully determined by the evolution of the radiation energy density.

$$\Omega_{\mathrm{m}} = 0, \qquad \Omega_{\mathrm{rad}} = 1, \qquad \Omega_{\Lambda} = 0, \qquad k = 0$$

The FRW equations reduce to:

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 = \frac{8\pi G\rho_0}{3}\frac{1}{a^2}$$

Which gives an expansion factor of:

$$\Rightarrow a(t) = \left(\frac{t}{t_0}\right)^{1/2}$$

The age of the Expansion Radiation Dominated Universe is:

$$t_0 = \frac{1}{2}\frac{1}{H_0}$$

5.3.4 De Sitter Expansion

The De Sitter Expansion is a model dominated by the cosmological constant, thought to correspond to dark energy in our universe or the inflation field in the early universe.

$$\Omega_{\mathrm{m}} = 0, \qquad \Omega_{\mathrm{rad}} = 0, \qquad \Omega_{\Lambda} = 1, \qquad k = 0$$

The FRW equations reduce to:

$$\Omega_{\Lambda} = \frac{\Lambda}{3{H_0}^2} \Rightarrow H_0 = \sqrt{\frac{\Lambda}{3}}$$

$$\dot{a}^2 = \frac{\Lambda}{3}a^2 \quad \Rightarrow \quad \dot{a} = H_0 a$$

Which gives an expansion factor of:

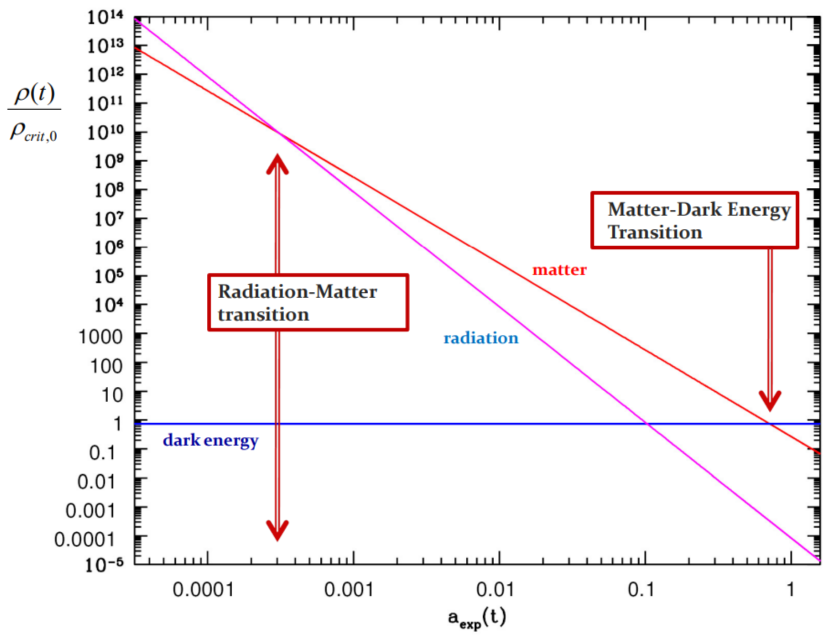
$$\Rightarrow a(t) = e^{H_0(t-t_0)}$$

The age of the Expansion Radiation Dominated Universe is:

$$t_0 = \infty$$

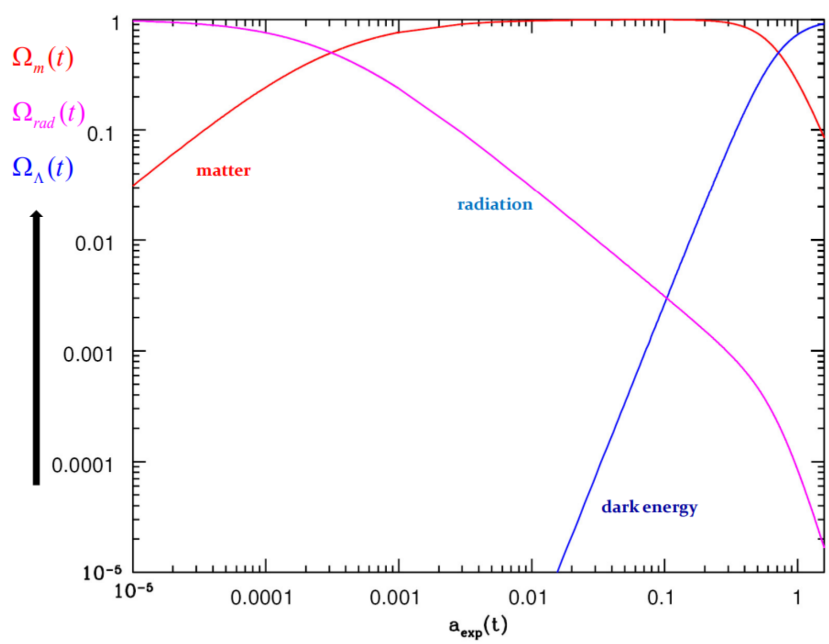
5.4 Evolving Cosmic Composition

5.4.1 Density Evolution Cosmic Components

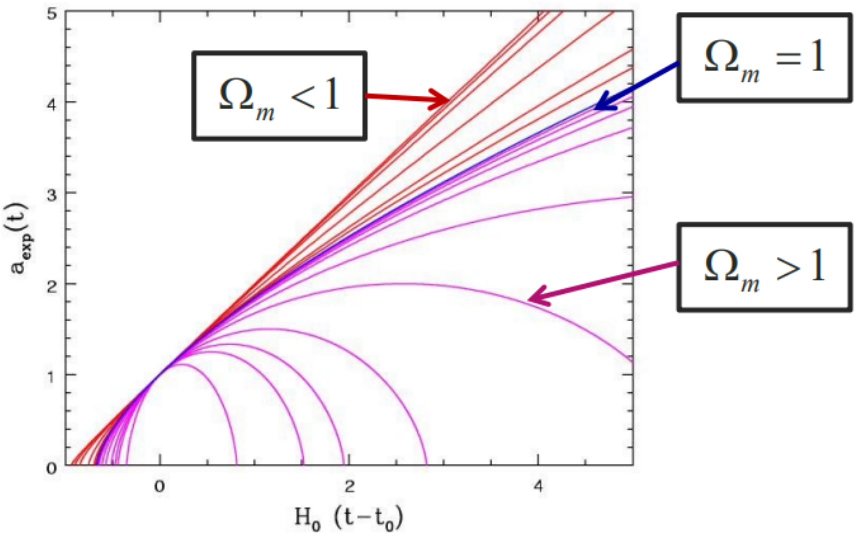


y-axis here is NOT Ω.

5.4.2 Evolution Cosmic Density Parameter



5.4.3 Matter Dominated Universes



5.5 Cosmological Transitions

5.5.1 Dynamical Transitions

Because radiation, matter, dark energy (and curvature) of the Universe evolve differently as the Universe expands, at different epochs the energy density of the Universe is alternately dominated by these different ingredients.

As the Universe is dominated by either radiation, matter, curvature or dark energy, the cosmic expansion a(t) proceeds differently.

We therefore recognize the following epochs:

- Radiation-dominated era (after Big Bang)
- Matter-dominated era (10 000 into Big Bang)
- Curvature-dominated expansion (not in our Universe)
- Dark energy dominated epoch (recently/now)

The different cosmic expansions at these eras have a huge effect on relevant physical processes.

Radiation Density Evolution: $$\rho_{\text{rad}}(t) = \frac{1}{a^4}\rho_{rad,0}$$

Matter Density Evolution: $$\rho_m(t) = \frac{1}{a^3}\rho_{m,0}$$

Curvature Evolution: $$\frac{kc^2}{R(t)^2} = \frac{1}{a^2}\frac{kc^2}{R_0^2} = \frac{1}{a^2}\,(1-\Omega_0)$$

Dark Energy Evolution: $$\rho_\Lambda(t) = cst. = \rho_{\Lambda 0}$$

5.5.2 Radiation-Matter Transition

Radiation energy density decreases more rapidly than matter density: this implies radiation to have had a higher energy density before a particular cosmic time:

$$\frac{\rho_{m,0}}{a^3} = \frac{\rho_{rad,0}}{a^4} \Rightarrow a_{rm} = \frac{\Omega_{\text{rad},0}}{\Omega_{m,0}}$$

$a < a_{rm}$	Radiation dominance
$a > a_{rm}$	Matter dominance

5.5.3 Matter-Dark Energy Transition

While matter density decreases due to the expansion of the Universe, the cosmological constant represents a small, yet constant, energy density. As a result, it will represent a higher density after

$$\frac{\rho_{m,0}}{a^3} = \rho_{\Lambda,0} \Rightarrow a_{m\Lambda} = \sqrt[3]{\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}}$$

$a < a_{m\Lambda}$	Matter dominance
$a > a_{m\Lambda}$	Dark Energy dominance

In a flat Universe we get

$$a_{m\Lambda} = \sqrt[3]{\frac{\Omega_{m,0}}{1 - \Omega_{m,0}}}$$

This transition was very recent:

$$\left.\begin{matrix} \Omega_{\Lambda,0} = 0.27 \\ \Omega_{m,0} = 0.73 \end{matrix}\right\} \quad a_{m\Lambda} = 0.72$$

(remember currently $a = 1$, by definition).

A more appropriate characteristic transition is that at which the deceleration turns into acceleration:

$$a^{\dagger}_{m\Lambda} = \sqrt[3]{\frac{\Omega_{m,0}}{2\Omega_{\Lambda,0}}} = \sqrt[3]{\frac{\Omega_{m,0}}{2\,(1 - \Omega_{m,0})}} = 0.57$$

5.5.4 Evolution Cosmological Density Parameter

Limiting ourselves to a flat Universe (and discarding the contribution by and evolution of curvature): to appreciate the dominance of radiation, matter and dark energy in the subsequent cosmological eras, it is most illuminating to look at the evolution of the cosmological density parameter of these cosmological components:

$$\Omega_m(t) = \frac{\Omega_{m,0}a^4}{\Omega_{rad,0} + \Omega_{m,0}a + \Omega_{\Lambda,0}a^4}$$

From the FRW equations, one can infer that the evolution of Ω goes like (for simplicity, assume matter-dominated Universe),

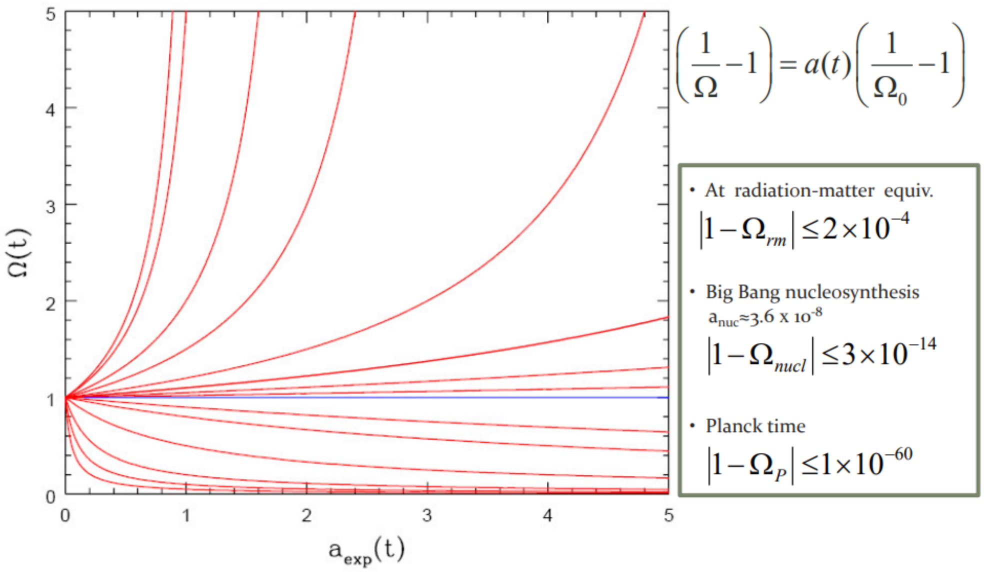
$$\left(\frac{1}{\Omega} - 1\right) = a(t)\left(\frac{1}{\Omega_0} - 1\right)$$

This equations directly shows that

$$a \downarrow 0 \Rightarrow \Omega \to 1$$

This implies that the early Universe was very nearly flat.

5.5.5 Flatness Evolution



5.6 Concordance Universe

5.6.1 Concordance Universe Parameters

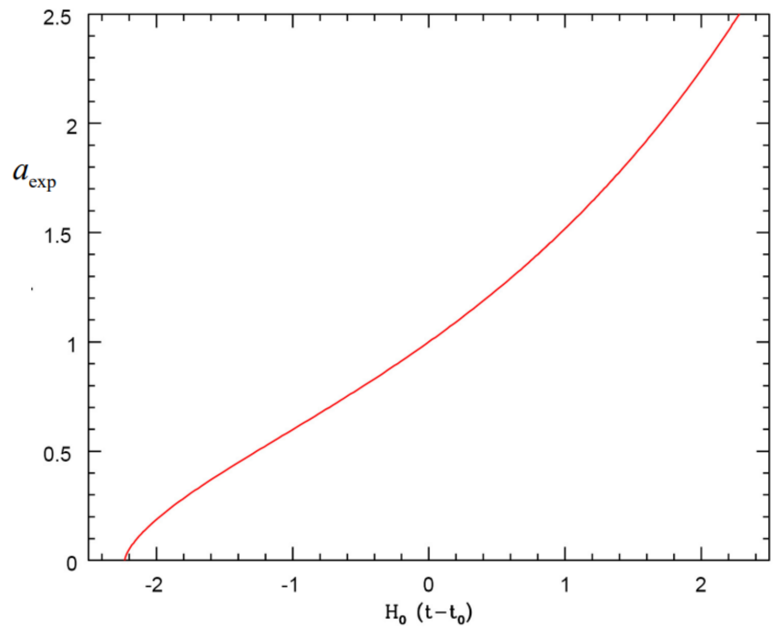
Hubble Parameter	$H_0 = 71.9 \pm 2.6$ km s^{-1} Mpc^{-1}
Age of the Universe	$t_0 = 13.8 \pm 0.12$ Gyr
Temperature CMB	$T_0 = 2.725 \pm 0.001$ K
Matter	$\Omega_\mathrm{m} = 0.27$
Baryonic Matter	$\Omega_\mathrm{b} = 0.0456 \pm 0.0015$
Dark Matter	$\Omega_\mathrm{dm} = 0.228 \pm 0.013$
Radiation	$\Omega_\mathrm{rad} = 8.4 \times 10^{-5}$
Photons (CMB)	$\Omega_\gamma = 5 \times 10^{-5}$
Neutrinos (Cosmic)	$\Omega_\nu = 3.4 \times 10^{-5}$
Dark Energy	$\Omega_\Lambda = 0.726 \pm 0.015$
Total	$\Omega_\mathrm{tot} = 1.0050 \pm 0.0061$

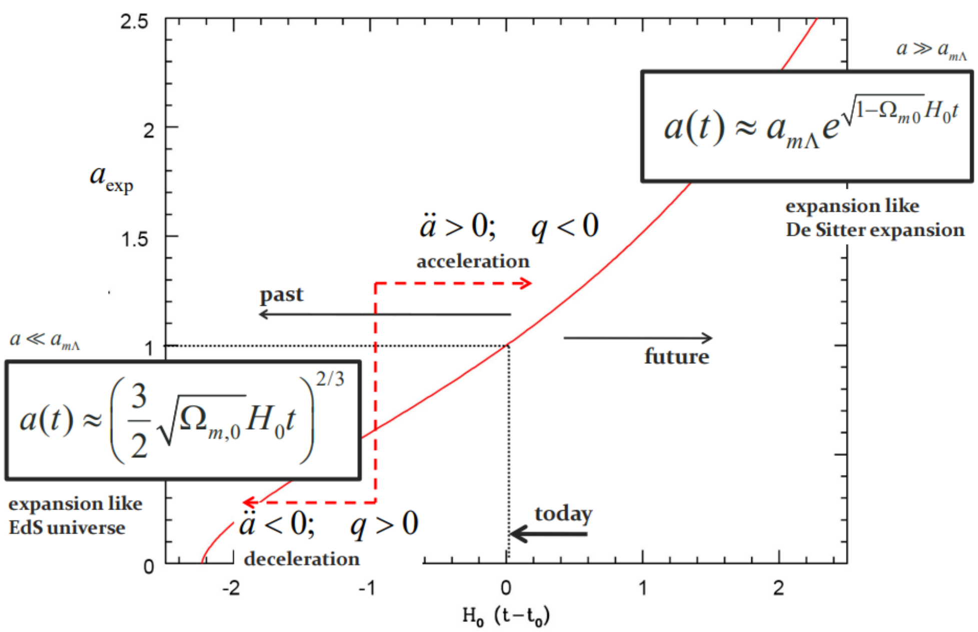
5.6.2 Concordance Expansion

The expansion of our Universe is given by:

$$H_0 t = \frac{2}{3\sqrt{1-\Omega_{m,0}}} \ln\left\{\left(\frac{a}{a_{m\Lambda}}\right)^{3/2} + \sqrt{1+\left(\frac{a}{a_{m\Lambda}}\right)^{3}}\right\}$$

This is plotted here:





We can recognize two extreme regimes:

$a \ll a_{m\Lambda}$, very early times, matter dominates the expansion, $\Omega_\mathrm{m} \approx 1$, this gives an **Einstein-de Sitter expansion**:

$$a(t) \approx \left(\frac{3}{2}\sqrt{\Omega_{m,0}}H_0 t\right)^{2/3}$$

$a \gg a_{m\Lambda}$, very late times, matter has diluted to oblivion, $\Omega_\mathrm{m} \approx 0$, this gives a **de Sitter expansion** driven by dark energy:

$$a(t) \approx a_{m\Lambda} e^{\sqrt{1-\Omega_{m0}}H_0 t}$$

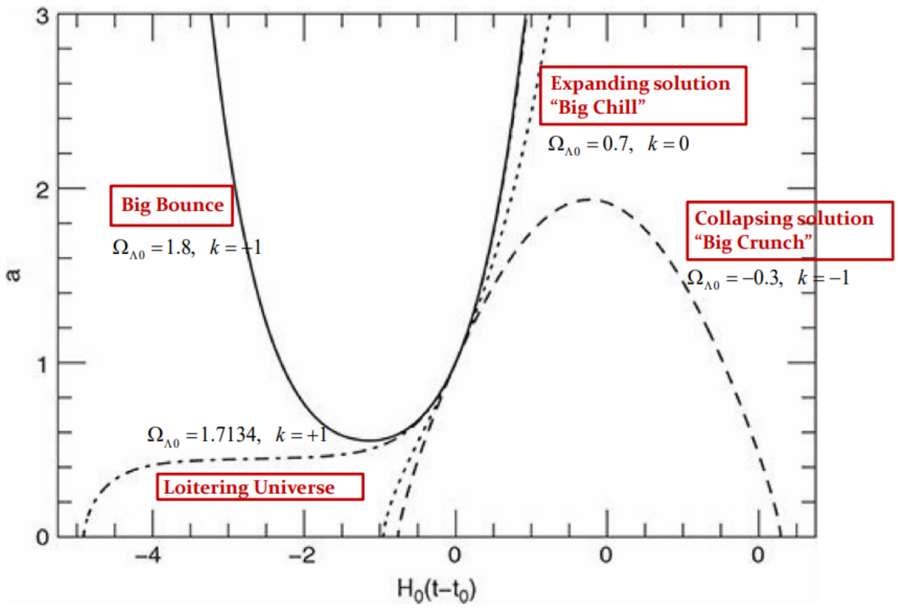
5.7 Expansion Histories

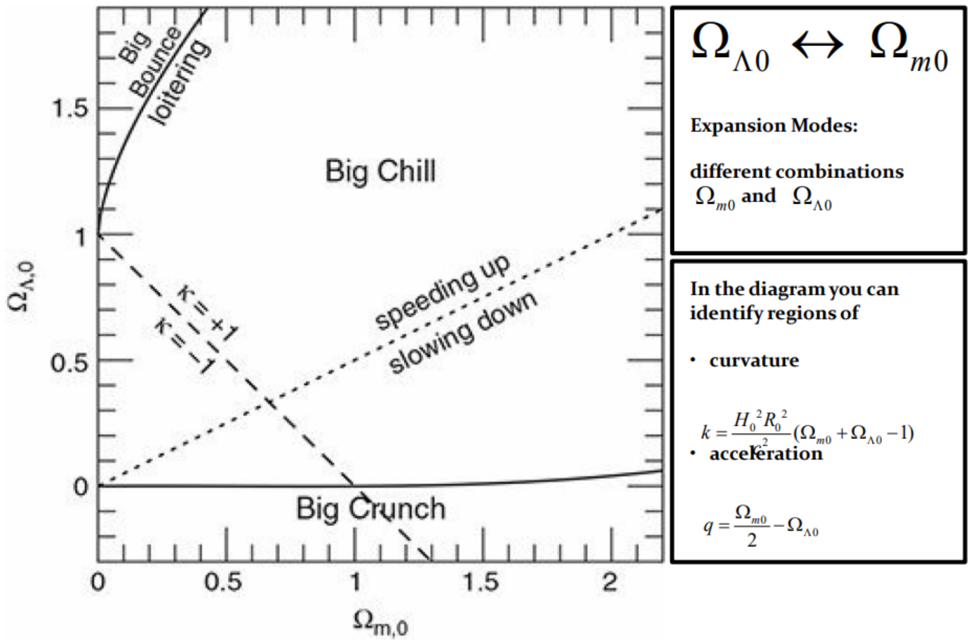
5.7.1 Cosmological Evolution Modes

It is interesting to inspect the possible expansion histories for generic FRWL cosmologies with matter & cosmological constant.
The expansion histories entirely determined by 2 parameters: matter density, $\Omega_{m,0}$ and the cosmological constant, $\Omega_{\Lambda,0}$.

We recognize 4 (qualitatively) different and possible modes of cosmic evolution:

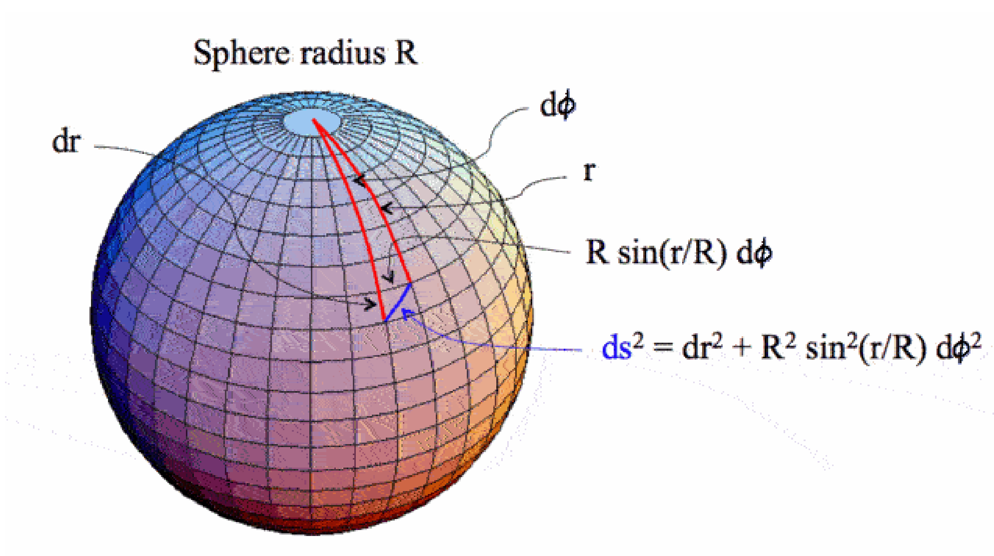
- Bouncing universe
- Collapsing universe "Big Crunch"
- Loitering universe
- Expansion (only) universe





6 Curved Cosmos

6.1 Spherical Space Distances



$$\left.\begin{array}{l} r = R\theta \\ dr = Rd\theta \end{array}\right\} ds^2 = dr^2 + R^2 \sin^2\left(\frac{r}{R}\right) d\phi^2$$

This of course leads to the Robertson-Walker metric.

6.2 Conformal Time

Proper time τ comes from the Robertson-Walker metric:

$$\begin{aligned} c^2 d\tau^2 &= ds^2 = c^2dt^2 - a(t)^2\left\{dr^2 + R_c^2 S_k^2\left(\frac{r}{R_c}\right)d\psi^2\right\} \\ d\psi^2 &= d\theta^2 + \sin^2\theta d\phi^2 \end{aligned}$$

To account of the expansion of the Universe, we divide the proper time by the radius of the curvature:

$$\begin{aligned} d\eta_\tau^2 = \frac{c^2d\tau^2}{R^2} &= \frac{c^2dt^2}{R^2} - \left\{d\left(\frac{r}{R_c}\right)^2 + S_k^2\left(\frac{r}{R_c}\right)d\psi^2\right\} \\ &= d\eta^2 - \left\{dr'^2 + S_k^2\left(r'\right)d\psi^2\right\} \end{aligned}$$

The **Conformal Time** is given by:

$$\eta(t) = \int_0^t \frac{cdt}{R(t)}$$

7 Observational Cosmology

7.1 Cosmic Redshift

Suppose you live in an expanding Universe, looking at an object far away in the Universe, which emits an characteristic wavelength $\lambda_{\text{em}} = \lambda_0$. This radiation travels through the Universe and due to the finite velocity of light and the expansion of the Universe the waveform has been stretched out. When you observe the radiation it has changed to λ_{obs}. These two wavelengths are related by:

$$\lambda_{\text{obs}} = \frac{a\left(t_{\text{obs}}\right)}{a\left(t_{\text{em}}\right)}\lambda_0$$

This gives the redshift:

$$z \equiv \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}}$$

Combining this (and concluding $a\left(t_{\text{obs}}\right) = 1$) gives:

$$\left.\begin{aligned} \lambda_{\text{em}} &= \lambda_0 \\ \lambda_{\text{obs}} &= \frac{a\left(t_{\text{obs}}\right)}{a\left(t_{\text{em}}\right)}\lambda_0 \end{aligned}\right\} \Longrightarrow 1 + z = \frac{1}{a}$$

Note: this is **not** Doppler shift, this is not the compression of waves due to differences in velocity, this purely comes from the expansion of the Universe.

7.2 Cosmic Time Dilation

In an (expanding) space with Robertson-Walker metric, light travels with:

$$ds = 0: \quad c^2dt^2 - a(t)^2dr^2 = 0 \Rightarrow cdt = a(t)dr: \quad \frac{dt}{a(t)} = cst$$

Imagine you observe a time interval Δt_e, when observed this time interval has become:

$$\Delta t_{obs} = \frac{\Delta t_e}{a\left(t_e\right)}$$

This is the **Cosmic Time Dilation**.

Evidence Cosmic Time Dilation: light curves of supernovae (exploding stars) have a characteristic time interval over which the supernova rises and then dims, with high redshifts this characteristic time interval has a systematic shift.

7.3 Hubble Expansion

7.3.1 Interpreting Hubble Expansion

Cosmic Expansion manifests itself in the in a recession velocity which linearly increases with distance, this is the same for any galaxy within the Universe.
There is no centre of the Universe: would be in conflict with the Cosmological Principle.

7.3.2 Hubble Expansion

The Hubble law is given by:

$$v = Hr$$

7.3.3 Deformation Cosmic Volume Element

The evolution of a fluid element on its path through space may be specified by its velocity gradient:

$$\frac{1}{a}\frac{\partial v_i}{\partial x_j} = \frac{1}{3}\theta\delta_{ij} + \sigma_{ij} + \omega_{ij}$$

in which:

θ: velocity divergence
 $\Rightarrow$ contraction/expansion

σ: velocity shear
 $\Rightarrow$ deformation

ω: vorticity
 $\Rightarrow$ rotation of element

The Universe only contains the first, a velocity divergence H.

Anisotropic Relativistic Universe Models. The Bianchi I-IX Universe models can expand anisotropically. They have to be characterized by at least 3 Hubble parameters (expansion rate different in different directions). There are only marginal claims that indicate the possibility on the basis of CMB anisotropies.

Global Hubble Expansion. Observations over large regions of the sky, out to large cosmic depth confirm that the Hubble expansion offers a very good description of the actual Universe and that the Hubble expansion is the same in whatever direction you look: isotropic.

Hubble flow is given by:

$$H = \frac{1}{3}\nabla \cdot v$$

7.4 Distance Measure

In an (expanding) space with Robertson-Walker metric,

$$ds^2 = c^2dt^2 - a(t)^2\left\{dr^2 + R_c^2 S_k^2\left(\frac{r}{R_c}\right)\left[d\theta^2 + \sin^2\theta\, d\phi^2\right]\right\}$$

there are several definitions for distance, dependent on how you measure it.
They all involve the central distance function, the **RW Distance Measure**,

$$D(r) = R_c S_k\left(\frac{r}{R_c}\right)$$

7.4.1 RW Redshift-Distance

Observing in a FRW Universe, we locate galaxies in terms of their redshift z. To connect this to their true physical distance, we need to know what the coordinate distance r of an object with redshift z. Light propagation in a RW metric (curved space):

$$\left.\begin{array}{l} ds^2 = 0 \\ d\psi^2 = 0 \end{array}\right\} \Rightarrow cdt = -a(t)dr$$

Note:

- light propagation is along radial lines
- the "-" sign is an expression for the fact that the light ray propagating towards you moves in opposite direction of radial coordinate r

After some simplification and reordering, we find

$$cdt = c\frac{da}{\left(\frac{da}{dt}\right)} = c\frac{da}{\dot{a}} = c\frac{da}{Ha}$$

$$R(t) = a(t)R_0 = \frac{R_0}{1+z}$$

$$\Downarrow$$

$$\frac{dR}{R} = -\frac{1}{1+z}dz$$

Then you get a equation for the Redshift-Distance:

$$dr = \frac{c}{H(z)}dz$$

In a FRW Universe, the dependence of the Hubble expansion rate $H(z)$ at any redshift z depends on the content of matter, dark energy and radiation, as well as its curvature. This leads to the following explicit expression for the redshift-distance relation,

$$dr = \frac{c}{H_0}\left\{(1-\Omega_0)(1+z)^2 + \Omega_{\Lambda,0} + \Omega_{m,0}(1+z)^3 + \Omega_{rad,0}(1+z)^4\right\}^{-1/2} dz$$

Matter-Dominated Universe. In a matter-dominated Universe, the redshift-distance relation is

$$dr = \frac{c}{H_0}\left\{(1-\Omega_0)(1+z)^2 + \Omega_0(1+z)^3\right\}^{-1/2} dz$$

from which one may find that

$$r = \frac{c}{H_0}\int_0^z \frac{dz'}{(1+z')\sqrt{1+\Omega_0 z'}}$$

7.4.2 Mattig's Formula

The integral expression

$$r = \frac{c}{H_0}\int_0^z \frac{dz'}{(1+z')\sqrt{1+\Omega_0 z'}}$$

can be evaluated by using the substitution:

$$u^2 = \frac{k(\Omega_0 - 1)}{\Omega_0(1+z)}$$

This leads to Mattig's formula:

$$D(z) = R_c S_k\left(\frac{r}{R_c}\right) = \frac{2c}{H_0}\frac{\Omega_0 z + (\Omega_0 - 2)\left\{\sqrt{1+\Omega_0 z} - 1\right\}}{\Omega_0^2(1+z)}$$

This is one of the very most important and most useful equations in observational cosmology *(don't need to know this by heart tho < 3)*.

In a low-density Universe, it is better to use the following version:

$$D(z) = R_c S_k\left(\frac{r}{R_c}\right) = \frac{c}{H_0}\frac{z}{1+z}\frac{1+\sqrt{1+\Omega_0 z}}{1+\sqrt{1+\Omega_0 z}+\Omega_0 z/2}$$

For a Universe with a cosmological constant, there is not an easily tractable analytical expression (a Mattig's formula). The comoving Distance r has to be found through a numerical evaluation of the fundamental dr/dz expression.

7.4.3 Distance-Redshift Relation, 2nd Order

For all general FRW Universe, the second-order distance-redshift relation is identical, only depending on the deceleration parameter q_0:

$$D(z) = R_c S_k\left(\frac{r}{R_c}\right) \simeq \frac{c}{H_0}\left(z - \frac{1}{2}(1+q_0)\,z^2\right)$$

q_0 can be related to Ω_0 once the equation of state is known.

7.5 Angular Diameter Distance

Imagine an object of proper size d, at redshift z, its angular size $\Delta\theta$ is given by

$$d = a(t) R_c S_k\left(\frac{r}{R_c}\right)\Delta\theta \Longrightarrow \Delta\theta = \frac{d(1+z)}{D} = \frac{d}{D_A}$$

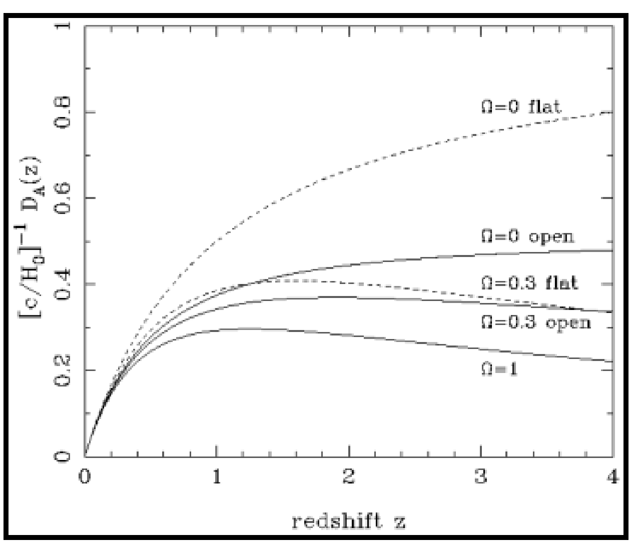
Angular Diameter distance:

$$D_A = \frac{D}{1+z}$$

In a matter-dominated Universe, the angular diameter distance as function of redshift is given by:

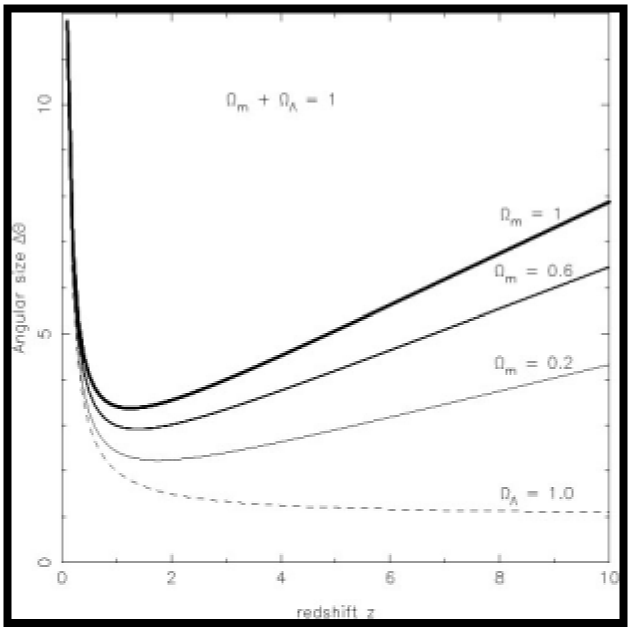
$$D_A(z) = \frac{1}{1+z} R_c S_k\left(\frac{r}{R_c}\right) = \frac{2c}{H_0}\frac{1}{\Omega_0^2(1+z)^2}\left\{\Omega_0 z + (\Omega_0 - 2)\left(\sqrt{1+\Omega_0 z} - 1\right)\right\}$$

This can be plotted:



The angular size $\theta(z)$ of an object of physical size ℓ at a redshift z displays an interesting behaviour. In most FRW universes is has a minimum at a medium range redshift – $z = 1.25$ in an $\Omega_m = 1$ EdS universe – and increases again at higher redshifts,

$$\theta(z) = \frac{\ell}{D_A}$$



7.6 Luminosity Distance

Imagine an object of luminosity $L(\nu_e)$, at redshift z, its flux density at observed frequency ν_o is

$$S\left(v_o\right) = \frac{L\left(v_e\right)}{4\pi D^2(1+z)} \quad \Rightarrow S_{bol} = \frac{L_{bol}}{4\pi D^2(1+z)^2} = \frac{L_{bol}}{4\pi D_L^2}$$

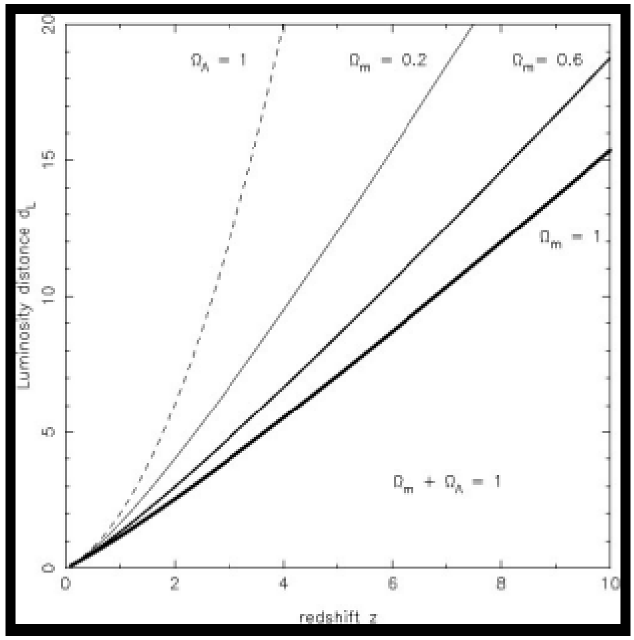
Luminosity distance:

$$D_L = D(1+z)$$

In a matter-dominated Universe, the luminosity distance as function of redshift is given by:

$$D_L(z) = (1+z)R_c S_k\left(\frac{r}{R_c}\right) = \frac{2c}{\Omega_0^2 H_0}\left\{\Omega_0 z + (\Omega_0 - 2)\left(\sqrt{1+\Omega_0 z} - 1\right)\right\}$$

This can be plotted:



7.7 Angular VS Luminosity Distance

The relation between the Luminosity and the Angular Diameter distance of an object at redshift z is sometimes indicated as the

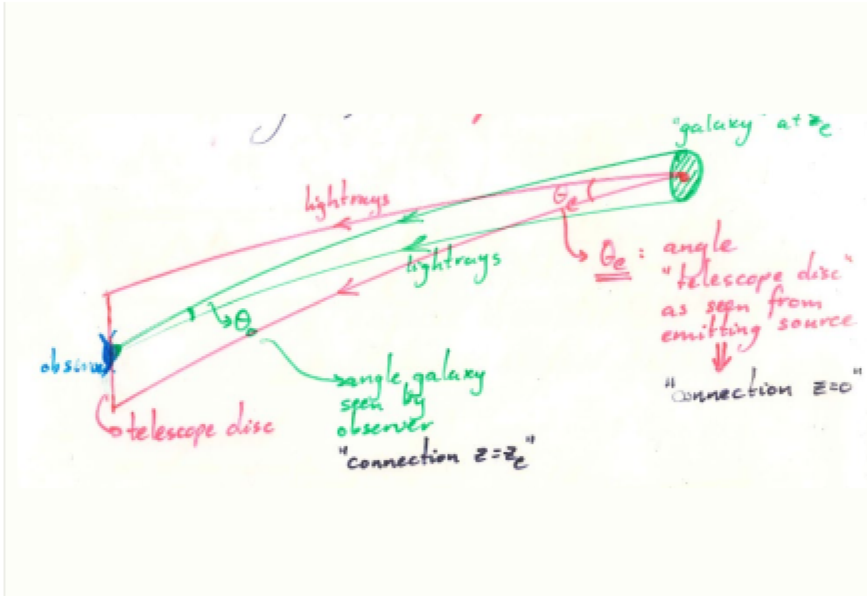
Reciprocity Theorem

The difference between these 2 fundamental cosmological measures stems from the fact that they involve "radial paths" measured in opposite directions along the lightcone, and thus are

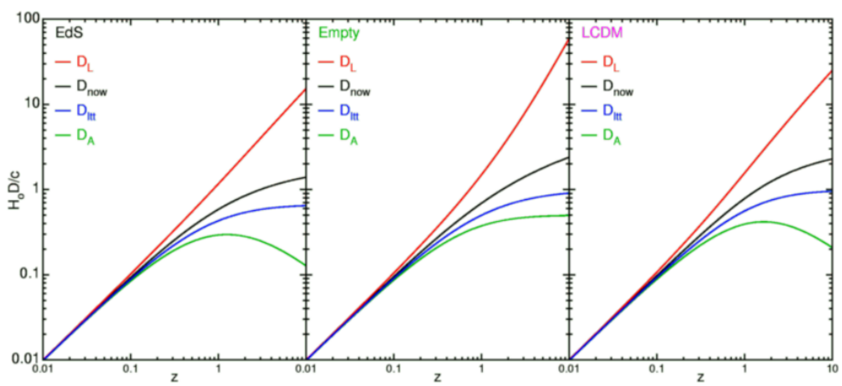
forward - luminosity distance
backward - angular diameter distance

wrt. expansion of the Universe

$$\left.\begin{array}{l} D_L = D(1+z) \\ D_A = D/(1+z) \end{array}\right\} \Rightarrow \frac{D_L}{D_A} = (1+z)^2$$



7.8 Cosmological Distances: Comparison



7.9 Cosmological Surface Brightness

Surface brightness of an extended object:

$$\sigma_o(z) = \frac{S_{bol}}{\Delta\Omega} = \frac{L_{bol}}{4\pi D_L^2}\frac{D_A^2}{\Delta A}$$

$$= \frac{L_{bol}}{4\pi\Delta A}\frac{D_A^2}{D_L^2}$$

$$= \sigma_{\text{int}}\frac{1}{(1+z)^4}$$

In a cosmological setting, the surface brightness $\sigma_0(z)$ of an extended object at redshift z diminishes very steeply as function of redshift z.
This makes it very difficult to study objects at high z in a similar detail as that in the local universe.

8 Universe Measured

8.1 Distance Measurements

Given the vast distances in the Universe, it is impossible to measure distances directly. Hence, we need to develop indirect methods that allow us to infer reliable estimates of the distances of objects.

One of the most practical means is based on the comparison between:

- observed brightness of an object **(apparent brightness)**
- intrinsic brightness of an object **(absolute brightness)**

8.1.1 Standard Candles

To determine distances in the Universe, astronomers identify objects of which they know the intrinsic brightness: **standard candles**.
Knowing the intrinsic luminosity/brightness Labs of a star/object, and measuring its apparent brightness, or flux S (light through per unit area), the distance D_L may simply be inferred from

$$S = \frac{L_{abs}}{4\pi D_L^2}$$

8.1.2 Cepheids: Period-Luminosity

To be able to determine cosmological distances, the reference Standard Candles need to be very bright objects/stars, whose intrinsic luminosity has been determined to high precision.

A particular type of variable stars, the Cepheid stars, have a brightness that varies as a result of their weeks long rhythmic pulsations
They have a characteristic relation between the period of their variation/pulsation and their intrinsic brightness. This is the so called **Period-Luminosity relation**.

As individual Cepheid stars are very bright - up to 100,000 times the Sun's luminosity, with masses in the order of $4 - 20M_\odot$ - they can be identified in other galaxies and the distance to those galaxies determined.

One of these Cepheids was used by Hubble to find the distance to the Andromeda galaxy!

8.2 Galaxy Velocities

8.2.1 Redshift

Velocity measurement: redshift/blueshift of radiation emitted by a source (galaxy, star).

Comparable to Doppler shift: the wavelength of radiation emitted by a source changes as it has a velocity towards or away from us:

towards us:

- towards shorter wavelength/higher frequency
- towards blue

away from us:

- towards larger wavelength/lower frequency
- towards red

Look at the spectrum (energy distribution of light) of the light emitted by a galaxy, red light has lower energy, blue light has higher energy.
Example: use prism to dissect light

In the spectrum of stars, you see a large number of lines:

- light/photons of specific energy/frequencies absorbed by atoms & molecules in the atmospheres of stars
- the frequencies of these spectral lines are fixed, by the quantum laws governing the structure and dynamics of atoms

8.2.2 Atoms & Spectral Lines

The energy of a photon is directly proportional to its frequency (ie. colour) and inversely proportional to its wavelength:

$$E = hv = \frac{hc}{\lambda}$$

The energy transitions go along with

towards higher level: absorption of photon with that specific energy

towards lower level: emission of photon with that specific energy

Energy of photon = frequency light

8.2.3 Galaxy Spectra & Cosmic Redshift

Galaxy spectra:

- the combined light of 100s billions of stars
- absorption lines mark the frequencies at which the atmospheres of the stars in the galaxy have absorbed light emitted by the stars

Galaxy redshift determination:

- identify (well-known and strong) spectral lines
- compare to rest wavelength, then determine z

$$z = \frac{\lambda - \lambda_0}{\lambda_0}$$

8.3 Hubble Expansion - Hubble Law

Lemaitre predicted linear relationship redshift – distance and inferred it from data and with this discovered the expanding Universe and knew this was the stretching of space. Nobody believed him, not even Einstein!

Finally, the ultimate evidence for an expanding Universe follows when Edwin Hubble describes his finding of a linear recession velocity – distance relation. This relation is now known as the **Hubble Law**

$$v_{rad} = cz = H_0 r$$

8.4 Type Ia Supernovae

Amongst the most energetic explosions in our Universe:

$$E \sim 10^{54} \text{ ergs}$$

During explosion the star is as bright as entire galaxy! (ie. 10^{11} stars).

Type Ia Supernovae are violent explosion of Carbon-Oxygen white dwarfs: embedded in binary, mass accretion from companion star. When nearing Chandrasekhar Limit ($1.38\,M_\odot$), electron degeneracy pressure can no longer sustain star. While contracting under its weight, carbon fusion sets in, powering a catastrophic deflagration or detonation wave, leading to a violent explosion, ripping apart entire star.

Because exploding stars have nearly uniform progenitor ($\sim 1.38\,M_\odot$ white dwarf), their luminosity is almost the same: $M \sim -19.3$: Standard Candle.

8.4.1 Supernovae SN1006

Supernova SN1006:

- brightness: $m = -7.5$
- distance: $d = 2.2$ kpc
- recorded: China, Egypt, Iraq, Japan, Switzerland, North America

8.4.2 Supernova Lightcurves

Relationship between the peak luminosity of a Type Ia supernova and the speed of luminosity evolution after maximum light.

Mark Phillips, on the basis of Calan/Tololo Supernova Survey, found that the faster a supernova fades after peak, the fainter its intrinsic peak luminosity. This reduces scatter in Hubble diagram, a heuristic relationship, as yet not theoretically "understood".

8.5 Cosmic Acceleration

Hubble Diagram high-z SNIa: distance vs. redshift z and $m - M$ vs. redshift z.

Determine:
- absolute brightness of supernova Ia
- from dimming rate (Phillips relation)

Measure:
- apparent brightness of explosion

Translates into:
- luminosity distance of supernova
- dependent on acceleration parameter q

These findings gave the acceleration of the Universe:

$$q_0 \approx \frac{\Omega_m}{2} - \Omega_v \approx -0.55$$

8.6 Cosmic Curvature Measured

8.6.1 Cosmic Microwave Background

Map of the Universe at Recombination Epoch (Planck, 2013):

- 379,000 years after Big Bang
- Subhorizon perturbations: primordial sound waves
- $\Delta T/T < 10^{-5}$

Measuring the Geometry of the Universe:

- Object with known physical size, at large cosmological distance
- Measure angular extent on sky
- Comparison yields light path, and from this the curvature of space

This gives the geometry of space.

For the object with known physical size, at large cosmological distance on can use the sound waves in the Early Universe!

- small ripples in primordial matter & photon distribution

- gravity:
  - compression primordial photon gas
  - photon pressure resists
- compressions and rarefactions in photon gas: sound waves
- sound waves not heard, but seen:
  - compressions: (photon) T higher
  - rarefactions: lower
- fundamental mode sound spectrum
  - size of "instrument":
  - (sound) horizon size last scattering
- Observed, angular size: $\theta \sim 1^o$
  - exact scale maximum compression, the "cosmic fundamental mode of music"

We know the redshift and the time it took for the light to reach us: from this we know the length of the legs of the triangle and the angle at which we are measuring the sound horizon.

$$v \approx \frac{c}{\sqrt{3}}$$
$$\ell \approx 200/\sqrt{1-\Omega_k}$$

The Cosmic Microwave Background Temperature Anisotropies: Universe is almost perfectly FLAT !

9 Dark Energy

Take-Home Facts

1. Strong evidence Accelerated Expansion
   - since supernova discovery, 100s SNIa observed over broader range redshifts
   - based solely upon supernova Hubble diagram, independent of General Relativity, very strong evidence expansion Universe accelerated recently
2. Dark energy as cause cosmic acceleration
   - within general relativity, accelerated expansion cannot be explained by any known form of matter or energy
   - it can be accommodated by a nearly smooth form of energy with large negative pressure, Dark Energy, that accounts for about 73% of the universe.
3. Independent evidence dark energy
   - Cosmic Microwave Background and Large Scale Structure data provide independent evidence, within context of CDM model of structure formation, that the universe is filled with a smooth medium accounting for 73% of the total energy content of the universe.
   - that came to dominate the dynamics of the universe once all observed structure had formed
4. Vacuum energy as dark energy
   - simplest explanation for dark energy is the energy associated with the vacuum
   - mathematically equivalent to a cosmological constant

  - However, most straightforward calculations of vacuum energy density from zero-point energies of all quantum fields lead to estimates which are a bit too large, in the order of $\sim 10^{120}$

5. Dark theories of Dark Energy
   - There is no compelling theory of dark energy
   - Beyond vacuum energy, man intriguing ideas: light scalar fields, additional spatial dimensions, etc.
   - Many models involve time-varying dark energy

6. New Gravitational Theories ?
   - alternatively, cosmic acceleration could be a manifestation of gravitational physics beyond General Relativity
   - however, as yet there is no self-consistent model for new gravitational physics that is consistent with large body of data that constrains theories of gravity.

9.1 Cosmic Horizons

Particle Horizon of the Universe: distance that light travelled since the Big Bang

Event Horizon of the Universe: the distance over which one may still communicate (this is decreasing)

10 Thermal History

10.1 FRW Thermodynamics

10.1.1 Adiabatic Cosmic Expansion

Important observation: the energy equation

$$\dot{\rho} + 3\left(\rho + \frac{p}{c^2}\right)\frac{\dot{a}}{a} = 0$$

is equivalent to stating that the change in internal energy

$$U = \rho c^2 V$$

of a specific co-expanding volume $V(t)$ of the Universe, is due to work by pressure:

$$dU = -pdV$$

Thus the Friedmann-Robertson-Walker-Lemaitre expansion of the Universe is an **adiabatic expansion**.

10.1.2 Thermal Evolution

An adiabatic expansion of the Universe has an implication for the Thermal History and for the temperature evolution of cosmic components.
For a medium with adiabatic index γ:

$$TV^{\gamma-1} = cst$$

Radiation (Photons)	$\gamma = \frac{4}{3}$	$T = \frac{T_0}{a}$
Monatomic Gas (hydrogen)	$\gamma = \frac{5}{3}$	$T = \frac{T_0}{a^2}$

10.2 Radiation & Matter

10.2.1 Cosmic Radiation

The Universe is filled with thermal radiation, the photons that were created in The Big Bang and that we now observe as the Cosmic Microwave Background (CMB).
The CMB photons represent the most abundant species in the Universe, by far!
The CMB radiation field is PERFECTLY thermalized, with their energy distribution representing the most perfect blackbody spectrum we know in nature. The energy density $u_\nu(T)$ is therefore given by the Planck spectral distribution,

$$u_\nu(T) = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT}-1}$$

At present, the temperature T of the cosmic radiation field is known to impressive precision,

$$T_0 = 2.725 \pm 0.001 \text{ K}$$

CMB is the most accurately measured Black Body Spectrum Ever!

With the energy density $u_\nu(T)$ of CMB photons with energy $h\nu$ given, we know the number density $n_\nu(T)$ of such photons:

$$n_\nu(T) = \frac{u_\nu(T)}{h\nu} = \frac{8\pi\nu^2}{c^3}\frac{1}{e^{h\nu/kT}-1}$$

The total number density $n_\gamma(T)$ of photons in the Universe can be assessed by integrating the number density $n_\gamma(T)$ of photons with frequency ν over all frequencies,

$$\begin{aligned} n_\gamma(T) &= \int_0^\infty n_\nu(T)d\nu = \\ &= \int_0^\infty \frac{8\pi\nu^2}{c^3}\frac{1}{e^{h\nu/kT}-1}d\nu = 60.4\left(\frac{kT}{hc}\right)^3 = 412 \text{ cm}^{-3} \end{aligned}$$

10.2.2 Baryon-Photon Ratio

Having determined the number density of photons, we may compare this with the number density of baryons, $n_b(T)$. That is, we wish to know the **photon-baryon ratio**,

$$\eta \equiv \frac{n_\gamma}{n_B} \qquad\qquad n_b = \frac{\rho_B}{m_p} = \frac{\Omega_B\rho_{\text{crit}}}{m_p}$$

The baryon number density is inferred from the baryon mass density. here, for simplicity, we have assumed that baryons (protons and neutrons) have the same mass, the proton mass $m_p \sim 1.672\times10^{-24}$ g. At present we therefore find

$$n_b = 1.12\times10^{-5}\,\Omega_b h^2 \text{ g cm}^{-3}$$
$$\Downarrow$$
$$\eta_0 = \frac{n_\gamma}{n_B} \approx 3.65\times10^7\,\frac{1}{\Omega_b h^2}\text{ g cm}^{-3}$$

We know that $\Omega_b \sim 0.044$ and $h \sim 0.72$, which gives:

$$\eta_0 = \frac{n_\gamma}{n_b} \approx 1.60\times10^9$$

The photon-baryon ratio in the Universe remains constant during the expansion of the Universe. This quantity is one of the key parameters of the Big Bang. The baryon-photon ratio quantifies the ENTROPY of the Universe, and it remains to be explained why the Universe has produced such a system of extremely large entropy!
The key to this lies in the very earliest instants of our Universe!

10.3 Hot Big Bang: Thermal History

10.3.1 Adiabatic Expansion

The Universe of Einstein, Friedmann & Lemaitre expands adiabatically. The energy of the expansion of the Universe corresponds to the decrease in the energy of its constituents. The Universe COOLS as a result of its expansion!

$$T(t) \propto 1/a(t)$$

10.3.2 Equilibrium Process

Throughout most of the universe's history (i.e. in the early universe), various species of particles keep in (local) thermal equilibrium via interaction processes:

$$\psi_i + \psi_j \rightleftarrows \chi_i + \chi_j$$

Equilibrium as long as the interaction rate Γ_{int} in the cosmos' thermal bath, leading to N_{int} interactions in time t,

$$\Gamma_{\text{int}} \quad \Rightarrow \quad N_{\text{int}} = \int \Gamma_{\text{int}}(t) dt$$

is much larger than the expansion rate of the Universe, the Hubble parameter $H(t)$:

$$\Gamma_{\text{int}} \gg H(t)$$

10.3.3 Reconstruction Thermal History Timeline

To work out the thermal history of the Universe, one has to evaluate at each cosmic time which physical processes are still in equilibrium. Once this no longer is the case, a physically significant transition has taken place. Dependent on whether one wants a crude impression or an accurately and detailed worked out description, one may follow two approaches:

Crudely:
Assess transitions of particles out of equilibrium, when they decouple from thermal bath. Usually, on crude argument:

$$\Gamma_{\text{int}} \gg H(t) \quad \Longrightarrow \quad \Gamma_{\text{int}} < H(t)$$

Strictly:
evolve particle distributions by integrating the Boltzmann equation

10.3.4 Thermal History: Interactions

Particle interactions are mediated by gauge bosons:

- photons for the electromagnetic force,
- W bosons for weak interactions,
- gluons for the strong force.

The strength of the interaction is set by the coupling constant, leading to the following dependence of the interaction rate Γ, on temperature T:

Mediated by massless gauge boson (photon):

$$\Gamma_{\text{int}} / H \sim \alpha^2 m_{\text{Pl}} / T$$

α : coupling strength

m_{Pl} : Planck mass

Mediated by massive gauge boson ($W^{+/-}$, Z^0):

$$\Gamma_{int}/H \sim G_x^2 m_{Pl} T^3$$

G_x : coupling strength

m_{Pl} : Planck mass

10.4 Hot Big Bang Eras

10.4.1 Cosmic Epochs

Planck Epoch		$t < 10^{-43}$ sec
Phase Transition Era	GUT transition Electroweak transition Quark-hadron transition	$10^{-43} < t < 10^{-5}$ sec
Hadron Era		t 10^{-5} sec
Lepton Era	Muon annihilation Neutrino decoupling Electron-positron annihilation Primordial nucleosynthesis	10^{-5} sec $< t <$ 1 min
Radiation Era	Radiation-matter equivalence Recombination & decoupling	1 min $< t < 379,000$ yrs
Post-Recombination Era	Structure & Galaxy formation Dark Ages Reionization Matter-Dark Energy transition	$t > 379,000$ yrs

10.5 History of the Universe in Four Episodes

On the basis of the complexity of the involved physics and our knowledge of the physical processes we may broadly distinguish four cosmic episodes:

10.5.1 (I): Planck Era

$$10^{-43} < t < 10^{-5} \text{ sec}$$

Origin of the Universe. The fundamental physics are totally unknown.

More Details. In principle, temperature T should rise to infinity as we probe earlier and earlier into the universe's history:

$$T \to \infty, \quad R \downarrow 0$$

However, at that time the energy of the particles starts to reach values where quantum gravity effects become dominant. In other words, the de Broglie wavelength of the particles become comparable to their own Schwarzschild radius.

10.5.2 (II): Very Early Universe

$$10^{-43} < t < 10^{-3} \text{ sec}$$

The fundamental physics are poorly known and speculative, but we have some ideas.

- Ω_{tot}: curvature/ flatness
- Ω_{b} (n_b/n_γ)
- 'exotic' dark matter
- primordial fluctuations

10.5.3 (III): Standard Hot Big Bang Fireball

$$10^{-3} < t < 10^{13} \text{ sec}$$

The fundamental microphysics are known very well.

- primordial nucleosynthesis
- blackbody radiation: CMB

10.5.4 (IV): Post (Re)Combination Universe

$$t > 10^{13} \text{ sec}$$

The fundamentals of the complex macrophysics are known and we're getting to know more and more of the complex interplay.

Structure formation: stars, galaxies, clusters, ...

10.6 Statistical Equilibrium

10.6.1 Maxwell-Boltzmann

Non-relativistic medium For statistical equilibrium, the Maxwell-Boltzmann distribution specifies for a temperature T, the density of particles:

- number density n_i
- particles with mass m_i
- statistical weight g_i
- chemical potential μ_i

$$n_i = g_i \frac{(2\pi m_i kT)^{3/2}}{(2\pi\hbar)^3} \exp\left\{\frac{\mu_i - m_i c^2}{kT}\right\}$$

10.7 Echo of the Big Bang: Recombination, Decoupling, Last Scattering

Before the "Recombination Epoch Radiation" and Matter are tightly coupled through Thomson scattering. The events surrounding "recombination" exist of THREE major (coupled, yet different) processes:

- **Recombination:** protons & electrons combine to H atoms
- **Decoupling:** photons & baryonic matter no longer interact
- **Last scattering:** meaning, photons have a last kick and go

10.7.1 Recombination

Once the temperature starts to drop below $T \sim 3000$ K, it becomes thermodynamically favorable to form neutral (hydrogen) atoms H (because the photons can no longer destroy the atoms):

$$p + e^- \rightleftarrows H + \gamma$$

This transition is usually marked by the word "recombination", somewhat of a misnomer, as of course hydrogen atoms combine just for the first time in cosmic history. It marks a radical transition point in the universe's history.

Statistical Equilibrium sets the density of electrons, protons and hydrogen atoms involved in the recombination process:

$$n_e = g_e \frac{(2\pi m_e kT)^{3/2}}{(2\pi\hbar)^3} \exp\left\{\frac{\mu_e - m_e c^2}{kT}\right\}$$
$$n_p = g_p \frac{(2\pi m_p kT)^{3/2}}{(2\pi\hbar)^3} \exp\left\{\frac{\mu_p - m_p c^2}{kT}\right\}$$
$$n_H = g_H \frac{(2\pi m_H kT)^{3/2}}{(2\pi\hbar)^3} \exp\left\{\frac{\mu_H - m_H c^2}{kT}\right\}$$

Taking along that for the chemical potentials

$$\mu_p + \mu_e = \mu_H$$

we find for the relation between the number densities

$$\frac{n_H}{n_e n_p} = \frac{g_H}{g_e g_p}\left(\frac{m_H}{m_e m_p}\right)^{3/2}\left(\frac{kT}{2\pi\hbar^2}\right)^{-3/2} \exp\left\{\frac{[m_p + m_e - m_H]\,c^2}{kT}\right\}$$

Then the

- small mass of the electron: $m_H/m_p \approx 1$
- binding energy of the hydrogen atom: $(m_e + m_p - m_H)\,c^2 = \chi = 13.6\text{eV}$
- statistical weights: $g_e = 2$, $g_p = 2$, $g_H = 4$

results in the Saha Equation,

$$\frac{n_H}{n_e n_p} = \left(\frac{m_e kT}{2\pi\hbar^2}\right)^{-3/2} \exp\left\{\frac{\chi}{kT}\right\}$$

which specifies the shifting ionization state as a function of shifting temperature T

Ionization. Photon number density in blackbody bath temperature T:

$$n_\gamma = \frac{2.404}{\pi^2}\left(\frac{kT}{\hbar c}\right)^3 = 0.243\left(\frac{kT}{\hbar c}\right)^3$$

Ionization fraction x: $X = \frac{n_p}{n_p + n_H}$ $\quad n_e = n_p = Xn$, $\quad n = n_p + n_H \simeq n_b$

Baryon-photon ratio η : $\eta = \frac{n_b}{n_\gamma} = \frac{n_p}{X n_\gamma}$

Proton number density at T : $n_p = 0.243 X\eta \left(\frac{kT}{\hbar c}\right)^3$

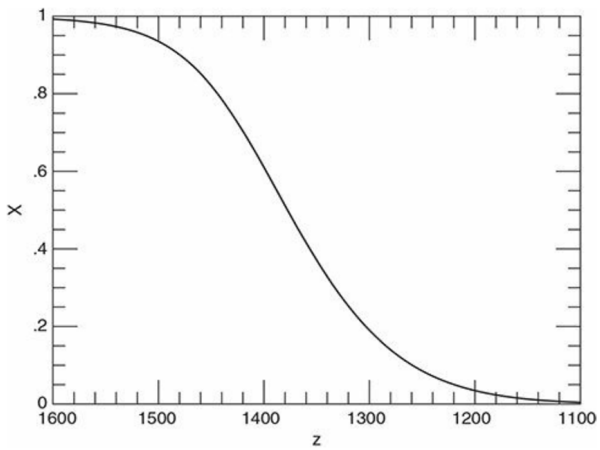
This gives the relation between temperature T and ionization fraction X:

$$\frac{1-X}{X^2} = 3.84\eta\left(\frac{kT}{m_e c^2}\right)^{3/2} \exp\left\{\frac{\chi}{kT}\right\}$$

Moment of recombination:

$$X = \frac{1}{2} \quad kT_{rec} \approx 0.323\text{eV} = 3740 \text{ K}$$

Ionization Evolution. Evolution of the previously mentioned shit.



But why 3000 K and not 3740 K??

Recombination Process not entirely trivial:

The ground state could be reached via Ly-α transition (2P-1S)
THIS DOES NOT WORK!
large abundance Ly-α photons $\Longrightarrow$ Ionization

Recombination in parts:
Via forbidden 2S − 1S transition =2-photon emission:
This takes a lil' longer, thus recombination occurs at 3000 K.

10.7.2 Decoupling

Decoupling:

$$\gamma + e^- \rightleftarrows \gamma + e^-$$

Thomson scattering:
Elastic scattering of photons off electrons

Cross-section:

$$\sigma_e = 6.65 \times 10^{-29}\ \mathrm{m}^2$$

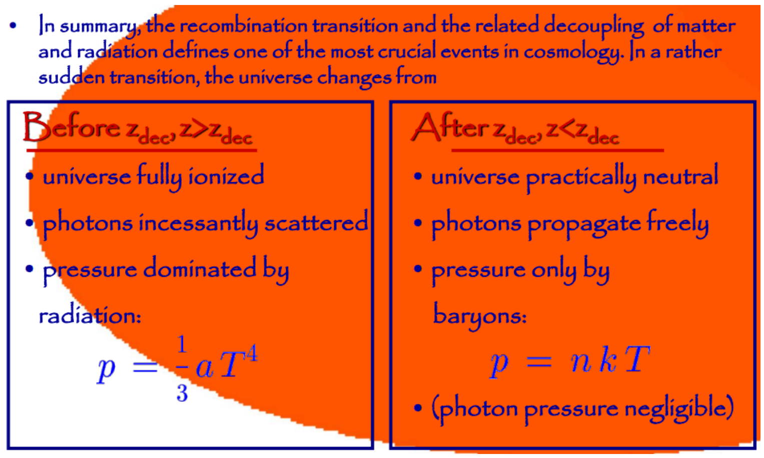
Mean free path:

$$\lambda = \frac{1}{n_e \sigma_e}$$

Interaction rate:

$$\Gamma = \frac{c}{\lambda} = n_e \sigma_e c$$

If fully ionized decoupling would happen at $T = 120$ K, but because recombination it happens earlier, at $T = 3000$ K



10.8 Origin CMB Photons

10.8.1 When were the CMB photons produced?

Most were produced when electrons & positrons annihilated each other, at $t \sim 1$ min, $z \sim 10^9$.
At the onset certainly not thermally distributed energies.
Photons keep on being scattered back and forth until $z \sim 1089$, the epoch of recombination.

10.8.2 How did they become a blackbody/thermal radiation field?

Thermal equilibrium (blackbody spectrum) of photons reached within 2 months after their creation. Blackbody Spectrum produced through three scattering processes.

- Compton scattering
- Free-free scattering
- Double Compton scattering

While Compton scattering manages to redistribute the energy of the photons, it cannot adjust the number of photons. Free-free scattering and Double Compton scattering manage to do so.

10.8.3 At which time were they scattered for the last time?

(in other words, what are we looking at ?)

10.9 First Three Minutes: Big Bang Nucleosynthesis

11 Inflation

11.1 Standard Big Bang: what it cannot explain

- **Flatness Problem:** the Universe is remarkably flat, and was even (much) flatter in the past
- **Horizon Problem:** the Universe is nearly perfectly isotropic and homogeneous, much more so in the past
- **Monopole Problem:** there are hardly any magnetic monopoles in our Universe
- **Fluctuations, seeds of structure:** structure in the Universe: origin

11.1.1 Flatness Problem

FRW Dynamical Evolution:
Going back in time, we find that the Universe was much flatter than it is at the present.
Reversely, that means that any small deviation from flatness in the early Universe would have been strongly amplified nowadays.
We would therefore expect to live in a Universe that would be almost $\Omega = 0$. Yet, we find ourselves to live in a Universe that is almost perfectly flat $\Omega = 1$.

11.1.2 Horizon Problem

Fundamental Concept for our understanding of the physics of the Universe:

- Physical processes are limited to the region of space with which we are or have ever been in physical contact.
- What is the region of space with which we are in contact ? Region with whom we have been able to exchange photons (photons: fastest moving particles)
- From which distance have we received light.
- Complication:
  - light is moving in an expanding and curved space
  - fighting its way against an expanding background

This is called the **Horizon of the Universe**.

In an Einstein-de Sitter Universe

$$R_{\mathrm{Hor}} = 3ct$$

The Horizon at recombination should span $\sim 1^o$, but CMB, the temperature is almost the same everywhere. For this everything should be in contact??

11.1.3 Structure Problem

CMB: almost featureless $\Delta T/T < 10^{-5}$.

11.2 INFLATION

Universe went through a phase transition; inflation.

At $\sim 10^{-36}$ sec. after Big Bang:
The Universe expands exponantially; a factor of 10^{60} in 10^{-34} sec.

Size of the visible Universe: Beginning of inflation: 10^{-15} times the size of an atom
End of inflation: diameter of a dime (stuiver)